DOCUMENT RESUME

ED 202 966 UD 021 465

TITLE	Minimum Learning Essentials: Mathematics. Fundamentals-Algebra, Geometry. Experimental Edition 0/3.
INSTITUTION	New York City Board of Education, Brooklyn, N.Y. Div. of Curriculum and Instruction.
PUB DATE	80
NOTE	325p.; For related documents, see UD 021 456-460, UD 021 463-471, and UD 021 474.
AVAILABLE FROM	Office of Curriculum Development and Support, Division of Curriculum and Instruction, 131 Livingston Street, Brooklyn, NY 11201 ($6.00; make checks payable to Publication Sales).
EDRS PRICE	MF01/PC13 Plus Postage.
DESCRIPTORS	*Algebra; Content Area Reading; Curriculum Guides; *Geometry; Grade 9; Grade 10; Grade 11; Grade 12; High Schools; *Mathematical Concepts; *Minimum Competencies; Study Skills
IDENTIFIERS	*New York City Board of Education

ABSTRACT

This "minimum teaching essentials" bulletin presents the minimum in basic skills and knowledge which must be taught to students in grades 9-12 in the subject areas of algebra, geometry, and mathematics fundamentals. Each subject area is prefaced with a section on work study skills to be used by students. (APM)

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0/3 EXPERIMENTAL EDITION

MINIMUM LEARNING ESSENTIALS

Mathematics

Fundamentals - Algebra - Geometry

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Foreword

High schools throughout the City of New York offer a wide range of enrichment and specialized courses to meet the needs of their students. Courses, both basic and specialized, vary from class to class, depending on the combination of challenge and creativity of the individuals involved. Within this rich variety, there is a need to establish minimum standards based on performance objectives within a subject area which can be used citywide to validate course content and student performance. This experimental series, Minimum Learning Essentials, has been developed to help teachers and supervisors identify the skills and knowledge which students in Grades 9-12 must acquire. The Minimum Learning Essentials, described here in a framework of performance objectives, represents the core of skills and knowledge students must acquire no matter where the course is taken.

The importance of this instructional framework is underscored by the need for increased articulation between and within departments in junior high schools and in high schools, and the rate of pupil mobility in our city.

It is our intention to extend the concept of Minimum Learning Essentials to all subject areas. The first series comprises the following:

ENGLISH/COMMUNICATION ARTS, Grades 9-12
Eight semesters emphasizing:

- Listening and Speaking
- Literature and Reading
- Writing, including Writing Conventions

MATHEMATICS

- Fundamentals of Mathematics (Grade 9)
- Algebra
- Geometry

SCIENCE

General Science (Grade 9)
Biology
Chemistry
Physics
Earth Science

SOCIAL STUDIES

American Studies
Economics
Global History

As the teachers and supervisors test the performance objectives in the coming year, it is vital that they evaluate these objectives for scope and appropriateness. Minimum Learning Essentials is experimental; feedback from teachers and supervisors using these materials is crucial to their future development.

Our society requires increasing skills and competencies of its citizens; the response of the educational community must be to develop the best possible standards for identifying and evaluating what we demand of our students. We can offer no less.

Charlotte Frank

Charlotte Frank
Executive Director

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How to Use This Bulletin

Minimum Learning Essentials is a set of suggested instructional objectives for high school mathematics. These instructional objectives are stated in terms of student performance. Written in a basic, elemental fashion, these objectives are designed to be tailored by individual teachers to meet their own instructional needs.

It is important to know:

a. what is meant by a performance objective;
b. how performance objectives relate to traditional instruction, i.e., to the developmental lesson; and
c. what options are available to teachers in shaping performance objectives.

The following distillation of principles and procedures for the development and use of performance objectives will help teachers and supervisors develop and use performance objectives in the mathematics curriculum.

Performance Objectives: A Definition

As used here, the term performance objective* describes what a student is expected to know or do. It defines precisely what knowledge, understanding, and skills the student will learn and demonstrate. In general, these objectives are stated at a simple degree of specificity and do not pretend to detail the conditions under which the learning is demonstrated nor the criteria for evaluating success. In spite of this simple degree of specificity (see chart, p. x), these objectives serve two purposes

1. They provide an emphasis on student performance (i.e., what is it the student is expected to do).

2. They serve as a starting point for the development of more intricate, complex, and detailed performance objectives.

* Readers familiar with the performance objectives movement will recognize the contributions of Benjamin Bloom and Robert Mager in the ensuing discussion. A more detailed treatment of these substantive issues and their effect on the New York City school system is presented in Performance Objectives: Experimental Edition available from the Division of Curriculum and Instruction. This brief discussion is offered to help teachers and supervisors get started working out their own performance objectives.

Performance objectives, then, always address themselves not to teacher behavior but to student behavior. They may be stated at low or high degrees of specificity. They are defined as: The intended terminal behavior on the part of students which is the intended result of an instructional activity.

Performance Objectives and Traditional Instruction

The teachers who prepared this material have paid considerable attention to the inclusion and sequencing of material, but have not provided complete lesson plans. A good lesson plan must include an aim, a motivation, development, applications, activities and summaries. To help the teacher devise this total kind of plan, there are references to specific pages in related curriculum bulletins in the right-hand column of each page.

Objectives in terms of student performance are not new to traditional instruction. In a developmental lesson, for example, the aims of the lesson (and frequently the ancillary aims) addressed themselves to student performance....but not always and not consistently. In one variety of the developmental lesson, the student aim addresses itself to the student behavior deemed desirable. Thus, a good point of departure for incorporating performance objectives into traditional instruction is the analysis of developmental lesson plans. Teachers will find that performance objectives may be utilized as aims, ancillary aims, student aims, medial summaries, and even as suggested follow-up and enrichment activities, without harming the integrity of the traditional lesson plan format. Their incorporation therein will have the effect of giving greater play to student results and learning competencies with the motivation, pivotal questions, and evaluation more precisely and consistently targeted onto the desirable and intended outcomes of the class.

Shaping Performance Objectives

Shaping performance objectives may be as simple or as complex as the situation requires. For the most part, in the present document we deal only with cognitive objectives (i.e., those which detail intellectual performances) leaving affective behaviors for a future treatment.

In writing performance objectives for mathematics skills, we try to avoid verbs such as "learn," "know," "appreciate," "think," etc., which are more difficult to observe and therefore more difficult to measure. However, through the careful selection and wording of instructional objectives the kind of thinking required is implied. If students can state a relationship, write a formula expressing the relationship, and use the relationship to solve particular types of problems, then it can be assumed that they understand the relationship at several levels of thinking. If the criteria for the evaluation part of the objective have been satisfied as well, then it can be assumed that the relationship has been mastered.*

Rather than focus in on the extremely complex myriad of intellectual competencies desirable on the part of students, three cognitive behaviors serve for this beginning discussion. They are:

I. knowledge (i.e., simple recall, recognition, identification, etc.);

II. understanding (i.e., comprehension, inference, translation, extrapolation, etc.);

III. utilization (i.e., application, analysis, synthesis, evaluation, etc.).

The "compression" of intellectual activity into these three broad categories is not arbitrary. Rather, it is presented on the basis of substantial classroom experience. Thus, before a student can really understand (II) fractions, there are certain terms and principles which he or she will have to know and recall (such as numerator, denominator, etc.). Thus cognitive behavior (simple recall of knowledge once achieved) permits behavior II, understanding; On the basis of ahievement in I and II, performance III, utilization, is possible, e.g., the calculation of discounts, interest rates, etc.

* Teachers who are involved in Mastery Learning are familiar with the use of instructional objectives classified according to levels of cognitive behavior based on Benjamin Bloom's Taxonomy of Educational Objectives.

The objectives for mathematics as presented cover this range of three cognitive behaviors. Teachers may choose to shift from one behavior level to another. For example, if the objective specifies a level II performance (infer, translate, extrapolate), it may be more productive for a certain class to recall prior knowledge and for the teacher to rephrase the performance objective in terms of knowledge (I).

In addition to emphasizing three cognitive behaviors, the objectives in mathematics are presented to assist teachers in the development of more complex, more individualized objectives (i.e., appropriate to the specific class or individual students). Professional experience has pointed out that performance objectives may be stated in at least four degrees. a continuum moving from general to specific

DEGREES OF SPECIFICITY

degree 1	degree 2	degree 3	degree 4
The statement of the "interior" intellectual behavior intended for learner achievement (*i.e.*, to identify).	The statement of an "overt" behavior on the part of the students from which the "interior" behavior can be inferred (*e.g.*, to write).	The delineation of the overt behavior detailing the "conditions" under which the behavior will be achieved (for example, to write from memory, without the text).	The delineation of the overt behavior detailing not only the conditions under which it will be achieved but also detailing the "criteria" for success (*e.g.*, to write from memory correctly four out of five times).

Any one of the objectives offered in this guide can be completed in various ways to suit the needs of individual teaching settings. For example, the performance objective to be found in Fundamentals of Mathematics, Section IV, "Using Formulas" (page 21):

> The student will be able to write a composite number as a product of two or more factors.

An analysis of this objective reveals that it calls for a cognitive Level III performance on the part of the student -- viz., utilization. As such, teachers can choose to accept it or to "move it back" to level I or II, depending on the achievement level of the class. If it is accepted as presented, teachers can then choose to give it more performance detail. As stated, it speaks to the "overt" behavior (i.e., to write) from which can be inferred the "interior" behavior, utilization. To move to degree 3 of statement specificity, the teacher, in detailing conditions of performance may choose to write:

> Given a composite number less than or equal to 100, the student will write it as the product of two or more factors without using a times table or calculator. (conditions underlined)

Moving on to the fourth and final degree of specificity of statement, should it be desirable, the teacher can add criteria of successful performance: Thus:

> Given a composite number less than or equal to 100, the student will write it as the product of two factors correctly in eight out of ten examples without using a times table or calculator. (criterion underlined)

Thus, this simple statement of overt behavior on the part of the student can be used as the basis for the development of an even more precise (i.e., conditioned and criterioned) statement of the same objective.

Getting Started

Given a clear notion of what a performance objective is, teachers are encouraged to incorporate them into their day-to-day planning and delivery of instruction. To begin, it is suggested that three cognitive behaviors

I. knowledge
II. understanding
III. utilization

be the initial focus. At the same time, four stages of statement specificity are available -- i.e.,

1. interior performance
2. overt performance
3. overt with conditions
4. overt with conditions and criteria.

The main purpose of the foregoing discussion is to enable teachers to begin to use performance objectives in their instruction. Only if teachers use performance-based developmental lessons will we know if the promise of performance objectives can be realized with our students.

Sources and Uses

Since these are minimum essentials, individual schools will want to include topics or objectives that have been omitted here. Teachers will add to them by providing review, explanations, illustrations, enrichments and alternative techniques of their own design wherever possible.

The New York State Regents Requirements for High School Mathematics served as a fundamental source for the instructional objectives for each course. The page references listed throughout are to standard and experimental New York City curriculum bulletins.

In studying the language used in these objectives, pay particular attention to the verb "state," which should be interpreted variously in different settings. It may mean "state orally" or "state in written form." It may mean "state in rigorous language" or "state informally."

Occasionally, objectives have been repeated at various places where it seemed that the review of certain skills was absolutely essential before going on. But certainly individual teachers will identify many more places for review.

Material included in parentheses should not be regarded as part of the objective per se, but is intended as a note to the teacher. For example, in the objective:

identify the subject of a formula
(in A = lw, A is the subject)

Acknowledgments

Minimum Learning Essentials is a project of the Division of Curriculum and Instruction, Charlotte Frank, Executive Director.

The overall concept for the design of the materials was reviewed by Nathan Quinones, Executive Director, and Charles Schonhaut, Superintendent of the Division of High Schools.

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OVERVIEW

MATHEMATICS

Mathematics is a basic study which has much to contribute to a changing society. Our pupils need basic mathematical knowledge which will give them the flexibility to function effectively as mathematically literate citizens and professionals.

It has become evident that no one mathematics course of study in the ninth year or in the succeeding years of high school can adequately serve the needs of all students. Therefore, different sequences of mathematics courses have been set up so that students may have an opportunity to benefit from a course which is most appropriate for their needs, interests, and abilities.

The General Mathematics course (usually taught in the ninth year) emphasizes fundamental skills used in real life situations. It also stresses the important aspects of non-computational mathematics, such as:

- the appreciation of the approximate nature of measurement
- the recognition and appreciation of geometric forms in the environment
- the ability to read and interpret graphs and charts
- the understanding and application of probability and statistics

Moreover, the General Mathematics course will provide meaningful mathematical experiences for the pre-algebra students to further their subsequent success in algebra or Course I of the three-year Sequential Mathematics Course. At the end of the General Mathematics course, all students will be eligible to take the RCT in mathematics.

For other high school students, the mathematics course best suited to their abilities is algebra which is based upon concepts and skills which were developed in previous grades and which will be extended in succeeding courses. The algebra curriculum emphasizes an understanding of structure, language and logic skills, and basic mathematical concepts as they relate to the set of integers, rational and irrational numbers and their application to such topics as

- the solution of equations
- graphing
- factoring
- ratio, proportion, indirect measurement
- the solution of verbal problems

The level of mathematical maturity of students must be considered. The approach and the rigor of mathematical reasoning introduced should be consistent with their capacity. Succeeding courses in geometry (usually the tenth year) and intermediate algebra/trigonometry (usually the eleventh year) will extend the basic ideas of this course and help students to see algebra as a postulational system.

While we are concerned with the use of deductive proof in arithmetic and algebra, the essence of both deductive and inductive reasoning is still best clarified in the study of geometry. Thus the feeling for mathematical structure gained through a postulational approach to arithmetic and algebra can make these studies more meaningful. Moreover, by the end of a one year course of study in geometry, the student should have a clear understanding of the importance of inductive and deductive reasoning in our understanding of the physical and philosophical world.

Therefore, this bulletin lists the topics and content areas along with corresponding instructional objectives for General Mathematics, (Fundamentals of Mathematics),* Algebra, and Geometry. These courses will satisfy the diploma requirements of the RCT and Regents examinations.

These courses represent the foundation of a high school mathematics program and will prepare students for more advanced courses such as intermediate algebra/trigonometry, calculus, computer science and other electives.

Notes on Programming Students

In order to reflect the ability of students who enter the General Mathematics course, it is suggested that those whose math score at the end of the eighth grade is less than 6.0 be programmed for a two-year sequence (Fundamentals of Mathematics 1,2,3,4). This course will cover the same instructional objectives as GM 1,2. The RCT should be administered to all students at the end of the General Mathematics or at the end of Fundamentals of Mathematics 1,2 and again as often as necessary.

* See notes on programming.

Students who fail the RCT may be programmed for an alternate RCT prep course which will have the same instructional objectives as General Mathematics, but will be presented with a different mathematical perspective.

Options for algebra students

In many schools students have an option of algebra 1,2 or algebra A,B,C. At the end of the algebra 1,2 sequence, the students are expected to take the Regents examination. However, for students who are having difficulty with algebra 1, there is the option of switching to algebra B. At the end of algebra B, the students would be expected to take the RCT exam.

Fundamentals of Mathematics

Work Study Skills

Work-study skills are those which students require in order to study, to work in class individually or in small groups, to do homework assignments. These skills are listed for each mathematical discipline in order to focus teachers' and supervisors' attention on elements which might be taken for granted. If we tell students to do more work, to study harder, we must be sure that the students know the skills that make their studying effective. It is hoped that an increased teacher awareness of the components of mathematical work-study skills will translate into increased skills and general competency for our students.

WORK STUDY SKILLS: FUNDAMENTALS OF MATHEMATICS

FOLLOWING DIRECTIONS

Taking notes from oral directions requiring several steps.

Applying a pattern for following written directions:

- Reading for an overview
- Rereading for specific steps
- Carrying out steps
- Checking for correct completion

Applying skills in following directions to the organization of homework, class work and examination papers (e.g., the display of work, the location of final answers, the calculation of answers to correct degrees of accuracy, etc.)

Translating directions concerning the rounding-off of answers into the expression of answers with the correct number of decimal places.

Working in small groups to solve a problem or invesigate a relationship.

Translating the directions "reduce" or "simplify" into the expression of final answers with the correct form.

Translating the directions "multiply," "add," "subtract," etc., into arithmetic examples utilizing the correct operations.

LOCATING INFORMATION

Interpreting and rephrasing questions.

Using a table of values of trigonometric ratios to locate the values for a specific angle, or, given the value of the ratio, to locate a specific angle.

Using a table of square roots to locate the square root or square of a given number.

Referring to notes for previous definitions, formulas, and techniques.

Finding the numerical information expressed in verbal problems.

Finding numerical information from diagrams.

Reading tables and charts to extract specific information called for verbally.

RECALLING INFORMATION

Stating previous formulas, definitions, and techniques.

Remembering the correct order of operations.

Remembering basic conversions between units within the English system and within the metric system.

INTERPRETING INFORMATION

From the statement of a verbal problem, choosing items of given information that would be needed for finding a solution

and

writing algebraic phrases, sentences which translate the relationships between quantities expressed verbally.

Expressing the product of a number of identical factors by using an exponent.

Reading numerical information from given diagrams on the number line.

Drawing a graph on a number line, given numerical information.

Interpreting a given diagram to calculate missing dimensions using the given dimensions of a figure.

Providing examples from real life of business, consumer, geometric or trade situations illustrating specific relationships between quantities.

Making judgments as to the relative sizes of quantities described in verbal problems or in diagrams.

ORGANIZING INFORMATION

Refining skills in note taking.

Maintaining lists of definitions, formulas and techniques.

Displaying work associated with the solution of numerical problems.

Making sketches based upon the conditions in verbal problems about geometric figures.

Preparing a table of values or summary chart.

Making a tree diagram to display the possible outcomes of an experiment.

MANIPULATING PHYSICAL MATERIALS

Measuring line segments with a ruler (English and metric).

Using a straightedge to draw line segments on number lines, geometric diagrams, charts, tables of values, etc.

Using models of units of area to determine the measure of surfaces.

Using other measuring instruments (e.g., thermometers, micrometers, scales, graduated beakers, etc.) to determine relationships between units of measure.

Using a string or tape measure to measure the circumference of a circle.

MINIMUM LEARNING ESSENTIALS

FUNDAMENTALS OF MATHEMATICS

The page references in this section are all coded FM1 or FM2, indicating corresponding lessons in the experimental bulletin Fundamentals of Mathematics I and II published in 1978 by the New York City Board of Education, Division of Curriculum and Instruction.

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
I. ROUNDING NUMBERS	The student will be able to ...	
A. Rounding Positive Whole Numbers	1. identify the place value of each digit in a given positive whole number containing up to ten digits.	
	2. read a given number containing up to ten digits.	
	3. write a number numerically, given the number verbally.	
	4. approximate sums and products (e.g., 18+240 = about 260; 22 x 31 = about 600).	
	5. name consecutive multiples of ten, one hundred and one thousand, starting with a given number (e.g., starting with 4300, name five consecutive multiples of 100: 4300, 4400, ...)	
	6. state two consecutive multiples of ten, one hundred or one thousand between which a given number is found (e.g., 2687 is between 2680 and 2690, 2600 and 2700, 2000 and 3000).	
	7. state that the numbers from zero to four are closest to zero, and the numbers from five to nine are closest to ten.	
	8. identify which consecutive multiple of ten, one hundred or one thousand a given number is closer to, and call that number the nearest ten, nearest hundred or nearest thousand.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	9. state that the digit used to round a number to the nearest ten, thousand, etc., is to the right of the specified place value.	
	10. round numbers with up to ten digits to any stated degree of accuracy, from nearest ten to nearest billion.	
B. Rounding Decimal Fractions	1. identify the place value of each decimal place in a given four-place decimal fraction.	
	2. read a given number with as many as four decimal places.	
	3. write a decimal fraction numerically, given the number verbally.	
	4. name the two one-cent amounts between which a given monetary value is found (e.g., $37.625 is between $37.62 and $37.63).	
	5. round a monetary amount to the nearest cent.	
	6. round a decimal fraction to the nearest hundredth.	
	7. state that the digit used to round off a decimal fraction to the nearest tenth, hundredth, or thousandth is to the right of the specified place value.	
	8. round any decimal number to the nearest tenth, hundredth, or thousandth.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
II. INTRODUCTION TO ALGEBRA	The student will be able to ...	
A. Symbols of Equality and Inequality	1. compare two whole numbers verbally: a is greater than b a is equal to b a is less than b	FM1: 1*
	2. use the symbols of equality or inequality to compare two whole numbers.	FM1: 1, 2
	3. state whether a sentence using $<$, $=$, or $>$ is true or false.	FM1: 2
	4. complete an open sentence including $<$ or $>$ with a number which makes it true.	
B. Positive Rational Number Line	1. construct a number line using a unit length with endpoints at 0 and 1, and extending it to the right to include all non-negative whole numbers.	
	2. Name the number to the left or right of a given number on the number line.	
	3. locate any positive whole number on the number line.	FM1: 3
	4. name the location of a given point on the number line with a positive whole number.	
	5. associate the relative size of numbers with the relative positions of their points on the number line (greater to the right, smaller to the left).	

The page references in this section are all coded FM1 or FM2, indicating corresponding lessons in the experimental bulletin Fundamentals of Mathematics I and II published in 1978 by the New York City Board of Education, Division of Curriculum and Instruction.

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	6. write a sentence to describe the order of two numbers on the number line using < or >.	FM1: 4
	7. write a sentence to describe the order of two lettered points on the line using < or >.	FM1: 3, 4
	8. state the rule: "If a is to the right of b, then a > b or b < a."	FM1: 3
	9. name points located between two integers on the number line with simple fractions.	FM1: 3
	10. identify equivalent fractions within a given set of fractions.	
	11. use < or > to write a sentence which describes the order of two ruler fractions (halves, quarters, eighths and sixteenths).	
	12. write a sentence using < or > to compare any positive fractions less than or equal to one (using only fractions with the same denominator).	
C. Ordering Rational Numbers with Unlike Denominators	1. determine the least common denominator, given two fractions with unlike denominators which are not relatively prime (e.g., 1/2 and 3/4, 1/12 and 3/8).	
	2. determine the least common denominator, given two fractions with relatively prime denominators.	
	3. use the identity property for multiplication to change the terms of fractions (e.g., 5/9 = ?/27).	FM1: 5
	4. rewrite two given fractions with unlike denominators as equivalent fractions with like denominators.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	5. write a sentence using $>$, $<$, or = to compare fractions with unlike denominators.	FM1: 6
D. Order of Operations	1. state that a numerical expression can have exactly one value.	
	2. apply the rules of the order of operations to phrases with two or three operations (using only whole numbers), without grouping symbols.	
	3. apply the rules of the order of operations to simple phrases with grouping symbols.	
E. Adding and Subtracting Positive Whole Numbers on the Number Line	1. add two whole numbers on a number line.	FM1: 7
	2. subtract two whole numbers on a number line.	FM1: 7
	3. use the terms sum and difference when referring to the above operations.	FM1: 7
	4. state and demonstrate the commutative and associative properties of addition on the number line.	FM1: 7
	5. state that a sum is equivalent to moving to the right on a number line.	FM1: 11
	6. state that a difference is equivalent to moving to the left on a number line.	FM1: 11
F. Adding and Subtracting Positive Rational Numbers on the Number Line	1. change an improper fraction into a mixed number and vice versa.	
	2. add two like proper fractions whose sum is less than one.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	3. add two like proper fractions whose sum is one.	
	4. add two like fractions whose sum is greater than one, and express the sum as a mixed number.	
	5. repeat 2, 3 and 4 for subtraction where the difference is always greater than or equal to zero. Do not use mixed numbers.	
	6. add or subtract positive rational numbers on the number line (in which denominators are less than ten).	FM1: 8
G. Multiplication and Division of Positive Whole Numbers on the Number Line	1. write any multiplication as a series of repeated additions.	FM1: 9
	2. find the product of two whole numbers less than ten on the number line.	FM1: 9
	3. demonstrate the commutative property of multiplication for whole numbers using the number line.	FM1: 9
	4. write any division as a series of repeated subtractions.	
	5. determine the quotient of a whole number by an exact divisor on the number line using repeated subtractions.	FM1: 10
	6. determine the quotient of a whole number by a smaller whole number which is not an exact divisor on the number line.	FM1: 10
H. Addition of Directed (Signed) Numbers on the Number Line	1. recognize ← and → as directional indicators to form directed numbers on a number line.	FM1: 12

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. add directed numbers using a number line where the result is 0 or to the right of 0.	FM1: 14
	3. extend the number line to the left of 0 to allow for movement in both directions.	FM1: 15
	4. define directed numbers (→) as positive numbers and (←) as negative numbers.	FM1: 15
	5. rename the points on the number line using positive and negative numbers.	FM1: 15
	6. add positive and negative numbers on the number line. Sums may be either positive or negative.	FM1: 15, 16
	7. apply the rules (+a) + (+b) = +a+b and (-a) + (+b) = -a+b.	
	8. define additive inverses as two numbers whose sum is 0.	
	9. write the additive inverse of a number.	
	10. demonstrate the associative and commutative properties of addition of integers by writing numerical sentences.	FM1: 19
	11. make an open sentence true using the set of signed numbers as the domain.	FM1: 23
	12. define absolute value of an integer as its distance from 0 on the number line.	
	13. state and apply the rules for adding two signed numbers.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
I. Subtraction of Signed Numbers	1. state that subtraction is the inverse operation of addition.	
	2. state that subtraction of $\overrightarrow{d}$ is the same as addition of $\overleftarrow{d}$.	
	3. state that subtraction of $\overleftarrow{d}$ is the same as addition of $\overrightarrow{d}$.	
	4. subtract signed numbers on a number line.	
	5. state the rule that subtracting a signed number is equivalent to adding its opposite.	
	6. apply the rule to subtract signed numbers.	
J. Ordering of Positive and Negative Numbers	1. Express the order of two positive integers in terms of their positions on the number line (a is to the left of b, so $a < b$).	FM1: 17
	2. use the symbols $<$ and $>$ for all signed numbers (i.e., $-2 > -6$ because -2 is to the right of -6).	FM1: 18
K. Multiplication of Signed Numbers	1. state the rules for multiplying two signed numbers	
	2. find the product of two signed numbers.	
	3. find the product of three signed numbers.	
	4. write an equation to illustrate the commutative property of multiplication for a given pair of signed numbers.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
L. Division of Signed Numbers	1. state the rules for dividing two signed numbers. 2. find the quotient of two signed numbers. 3. compute the other factor of a given signed number, given one factor. 4. write division in three ways (e.g., 6 divided by -3: $-3\overline{)6}$, $6 \div -3$, $\frac{6}{-3}$).	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
III. SOLVING EQUATIONS	The student will be able to ...	
A. Solving Equations With One Operation	1. distinguish between a phrase and a sentence.	
	2. state whether a sentence is an open sentence or not.	
	3. categorize a sentence as an equation or an inequality.	
	4. state whether a replacement for a variable makes an open sentence true or false (substituting integers and rationals).	FM2: 15
	5. use a given replacement set to find the solution set of an equation or inequality by trial and error. (Open sentences should be of the form $x + a = b$, $x - a = b$, $a x = b$, $\frac{x}{a} = b$.)	FM2: 16
B. Solving Equations Using Inverse Operations		
- Addition and Subtraction	1. state that addition and subtraction are inverse operations.	FM2: 18
	2, use the appropriate inverse operation to find the solution to equations of the form $x + a = b$ and $x - a = b$ (where a and b are integers, decimals and fractions).	FM2: 20
	3. check the solution of an equation.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
- Multiplication and Division	1. state that division and multiplication are inverse operations.	FM2: 22
	2. use the appropriate inverse operation to solve an equation of the form $a x = b$ and $\frac{x}{a} = b$ (where a and b are integers or decimals).	FM2: 23
	3. check the solutions of these equations.	
C. Solving Equations Using Addition and Subtraction Properties of Equality	1. state that $0 + x = x$.	
	2. demonstrate the addition and subtraction properties of equality with numerical sentences (e.g., if $6 + 1 = 7$, then $(6 + 1) + 3 = 7 + 3$).	FM2: 25
	3. solve equations using the addition and subtraction properties of equality.	FM2: 26
D. Solving Equations Using the Multiplication and Division Properties of Equality	1. state that $1 \cdot x = x$ for any number.	
	2. write the reciprocal of any non-zero integer.	
	3. show that the product of a number and its reciprocal is 1.	
	4. demonstrate the multiplication and division properties of equality in numerical sentences.	
	5. solve equations of the form $a x = b$ and $\frac{x}{a} = b$ using the multiplication and division properties of equality.	FM1: 27, 28

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
E. Solving Equations of the Form $a x + b = c$ Using Properties of Equality	1. state that the inverse of addition or subtraction must be used before the inverse of division or multiplication.	FM2: 139, 140
	2. solve and check equations of the form $a x + b = c$ and $a x - b = c$ using inverse operations, where a and b are positive.	FM2: 140
F. Solving Equations of the Form $\frac{ax}{b} = c$ Using Properties of Equality	1. write the reciprocal of a rational number.	
	2. use the multiplication property of equality to solve an equation of the form $\frac{ax}{b} = c$.	FM2: 143, 144
	3. check the solutions of the equations.	
G. Solving Equations of the Form $\frac{ax}{b} + c = d$ Using Properties of Equality	1. state the sequence of steps required for the solution of equations of the form $\frac{ax}{b} + c = d$ and $\frac{ax}{b} - c = d$ using inverse operations.	FM2: 146, 147, 148
	2. solve and check the solutions.	
H. Translating Verbal Expressions and Number Problems Into Algebraic Symbols	1. translate expressions referring to +, -, x, and ÷ into algebraic symbols, with particular attention to phrases such as "6 less than x" and "x subtracted from 10".	
	2. translate simple number problems into algebraic open sentences.	
	3. solve and check simple number problems.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
IV. USING FORMULAS		
A. Understanding Exponents	1. write a composite number as the product of two or more factors.	
	2. categorize numbers as prime or composite.	
	3. state that a perfect square can be written as the product of two equal factors.	
	4. express a perfect square as the second power of one of its factors.	
	5. find the value of a^n where a is a positive rational number and n is 2, 3, or 4.	
	6. evaluate numerical expressions applying the order of operations and including powers of whole numbers.	
B. Evaluating Formulas	1. describe several important uses of formulas in mathematics, science, and business.	FM2: 29, 31, 32
	2. identify the subject of a formula. (In A = l w, A is the subject.)	
	3. use the order of operations to evaluate the subject of a formula.	FM2: 33, 35, 36, 37, 38, 40, 41

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
V. PLANE GEOMETRY		
A. Classifying Triangles According to the Lengths of Their Sides	1. provide examples of geometric shapes in architecture, design, and nature.	FM1: 25
	2. label a line segment using two letters.	
	3. state the definition of a triangle.	FM1: 25
	4. label a triangle using three letters.	
	5. classify a given triangle as scalene, isosceles, or equilateral according to the number of sides of equal measure.	FM1: 25
B. Classifying Triangles According to the Measures of Their Angles	1. describe an angle as a rotation (as in the angle between the hands of a clock).	
	2. state that one complete rotation is divided into 360 units called degrees.	
	3. state that one-quarter of a rotation is a right angle.	
	4. state that one-half of a rotation is a straight angle.	
	5. state the definitions of an acute angle, a right angle, an obtuse angle, and a straight angle in terms of degrees.	
	6. select by sight acute, right, obtuse, and straight angles from a given set of angle diagrams.	
	7. state the rule that the sum of the measures of the angles of a triangle is 180°.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	8. use the rule to find the measure of the third angle of a triangle given the measures of the other two.	
	9. state that a triangle cannot include two right angles, two obtuse angles or a right angle and obtuse angle.	FM1: 28
	10. classify a given triangle as acute, right, or obtuse according to the measures of its angles.	FM1: 27
	11. classify a triangle according to sides and angles at the same time (e.g., right-isoscles, acute-scalene).	
	12. state that equilateral triangles are equiangular.	FM1: 27
	13. state that an isosceles triangle has two angles of equal measure.	FM1: 27
	14. name the parts of a right triangle.	FM1: 27
	15. identify the largest side in a right triangle as the hypotenuse.	FM1: 27
C. Classifying Quadrilaterals: Parallelogram, Rectangle, Rhombus, Square, Trapezoid	1. Define "quadrilateral."	FM1: 29
	2. provide examples of parallel lines in the real world (e.g., railroad tracks).	FM1: 29
	3. describe parallel lines as lines which are always the same distance apart.	
	4. identify the "special" quadrilaterals by their shapes (parallelogram, rectangle, rhombus, square, trapezoid).	FM1: 29

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	5. list the properties of sides and angles of a given type of quadrilateral.	FM1: 29
	6. name quadrilaterals whose sides and angles have given properties.	
	7. list properties of the diagonals of given types of quadrilaterals (especially bisecting each other, equal lengths, and forming right angles).	FM1: 29
D. Identifying Polygons of More Than Four Sides	1. state a definition of "regular polygon."	FM1: 31
	2. name regular polygons containing 5, 6, 8, 10 and 12 sides.	FM1: 31
	3. state the relationship between the angles of some polygons that allows them to tesellate or "fit" together. (The sum of the angles about a point $= 360^{\circ}$.)	FM1: 32
	4. state the relationship between the number of sides and the number of diagonals in a polygon.	FM1: 32
	5. state the relationship between the number of sides in a polygon and the number of triangles with a common vertex that may be formed in the polygon.	FM1: 33
E. Identifying Some Points and Lines in a Circle	1. define a circle as the figure formed by all the points a given distance from a given point.	FM1: 34
	2. define a circle as the figure formed by constructing a regular polygon with a "very large" number of sides.	FM1: 34
	3. define and draw a radius, a diameter, a chord, an arc, a sector.	FM1: 34

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	4. state the relationship between a radius and a diameter.	FM1: 34
	5. solve simple arithmetic problems relating diameters and radii, using fractions and decimals.	FM1: 35
	6. construct a circle using a compass.	
	7. lay off a given length on a line segment repeatedly, using a compass.	
	8. construct a regular hexagon, an equilateral triangle, and a square, all inscribed in a circle.	FM1: 35
F. Calculating the Perimeter of a Polygon	1. use a ruler (English and metric) to measure lengths, from several inches or centimeters to several feet or meters	FM1: 40
	2. define perimeter as the sum of the lengths of the sides of a polygon.	FM1: 36
	3. provide examples of the use of perimeter in the real world (e.g., fence, molding around a room, walking around a field, etc.)	
	4. use fractions and decimal numbers to calculate perimeters of polygons.	FM1: 37, 38
	5. use properties of triangles (isosceles, equilateral) and polygons when solving perimeter problems.	FM1: 38, 39, 42, 43
G. Calculating the Circumference of a Circle	1. define circumference as the distance around a circle.	FM1: 40
	2. measure circumferences of common objects using a string and a ruler, or a tape measure.	FM1: 40

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	3. calculate an approximate value of π by dividing the circumference of each object by its diameter.	FM1: 40
	4. state that the circumference of a circle is equal to 3.14 times its diameter.	
	5. calculate the circumferences of cirlces using $\frac{22}{7}$ or 3.14 for π, where the radii and diameters are fractions or decimals.	FM1: 41, 44

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
VI. CALCULATING AREAS		
A. Area of the Rectangle and Square	1. provide examples of the use of area in the real world (e.g., carpeting, lawn seed, floor tiles).	
	2. state that surface (area) must be measured with a square unit , and is different from perimeter which is measured in linear units.	FM1: 72
	3. find the areas of rectangles and squares by counting the number of boxes they enclose on a grid.	
	4. state that the area of a rectangle can be found indirectly, using $A = lw$.	FM1: 73
	5. state that the area of a square can be found by $A = s^2$.	
	6. calculate the areas of rectangles and squares using $A = lw$, $A = bh$ and $A = s^2$, and express the answers using square units.	FM1: 73, 74, 75
	7. use the formulas for the areas of rectangles and squares to calculate the base, height or side if the other quantities are given.	
	8. calculate the areas of more complex regions involving rectangles by dividing them into rectangles.	FM1: 76-78
B. Area of a Triangle	1. apply the formula $A = \frac{1}{2}bh$ to calculate the area of right triangles and express the area in square units.	FM1: 80

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. identify the base and height of any triangle, where the bottom side is the base.	
	3. use the formula A = ½bh to calculate the area of any triangle.	FM1: 81, 82
	4. use the formula A = ½bh to calculate the base or height of a triangle if the other quantities are given.	FM1: 82
	5. calculate the areas of more complex regions involving rectangles and triangles.	FM1: 84-86
C. Area of a Circle	1. state the formula used to find the area of a circle: $A = \pi r^2$ or $A = \pi \times r \times r$.	FM1: 87
	2. calculate the areas of various circles, given the diameter or the radius using whole numbers, decimals or fractions, and using an appropriate approximation of pi.	FM1: 87, 88

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
VII. SOLID GEOMETRY		
A. Identifying Prisms and Cylinders and Classifying Them According to the Shapes of Their Bases	1. identify plane and solid objects as rectangular, square and circular.	FM2: 107
	2. distinguish between drawings or objects which are plane or solid figures.	FM2: 107
	3. distinguish between prisms and cylinders.	FM2: 108
	4. state that a prism or cylinder is named by the shape of its base.	FM2: 107, 108
	5. identify given prisms and cylinders by the shapes of their bases.	
B. Volume of a Prism and a Cylinder	1. state that the space displaced by a solid object is different from length or area, and requires a cubic unit of measure.	FM2: 110
	2. provide examples of cubes (e.g., dice, ice, sugar).	
	3. use cubes (sugar, plastic, etc.) to measure the volume of a rectangular solid.	
	4. find the product of three whole numbers or combinations of whole numbers, fractions and decimals (apply the associative property of multiplication).	
	5. use the formula $V = lwh$ to calculate the volume of a rectangular prism, where l, w, and h are combinations of whole numbers, decimals and fractions.	FM2: 111

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	6. state that l x w is the area of the base of the prism, so that its volume is the product of the area of its base and its height.	FM2: 111
	7. calculate the volume of any prism using the formula V = area of base x height.	FM2: 112-113
	8. find the volume of a right-circular cylinder.	FM2: 114-115
C. Identifying Pyramids and Cones and Finding Their Volumes	1. distinguish between pyramids and prisms by recognizing that pyramids have triangular sides and prisms have rectangular sides.	FM2: 117
	2. name a pyramid by the shape of its base.	FM2: 119
	3. find the product of a fraction and a whole number.	
	4. find the product of a unit fraction and two or three whole numbers by reducing first, or by multiplying the whole numbers first.	
	5. use the formula V = $\frac{1}{3}$ lwh, or V = $\frac{1}{3}$ area of base x height to calculate the volume of a pyramid.	FM2: 118, 119, 120
	6. distinguish between cones (circular base) and pyramids (polygon base).	FM2: 121
	7. use the formula V = $\frac{1}{3}$ area of base x height to calculate the volume of a cone.	FM2: 122
	8. identify a given geometric solid as a prism, a cylinder, a pyramid, or a cone.	FM2: 123

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
VIII. RATIOS AND PROPORTIONS IN SIMILAR TRIANGLES		
A. Solving Proportions	1. compare two quantities with a ratio: a:b or $\frac{a}{b}$.	
	2. state that a proportion is an equation showing that two ratios are equal: a:b = b:c	
	3. identify an equation as a true proportion by reducing the fractions and comparing them.	
	4. use the proportion rule to test whether two fractions form a proportion. (In a proportion, the product of the means is equal to the product of the extremes. If $\frac{a}{b} = \frac{c}{d}$, then ad = bc.)	
	5. solve equations of the form ax = b.	
	6. use the proportion rule to solve proportions (e.g., $\frac{a}{b} = \frac{x}{c}$).	FM2: 160
B. Applying Proportions in Similar Triangles and Scale Drawings	1. describe situations in which a measurement of length or distance cannot be made directly.	FM2: 149-152
	2. state that triangles which have the same shape are called similar triangles.	FM2: 155
	3. state that when two triangles are similar, their angles can be made to coincide (corresponding angles are equal in measure), and their sides form equal ratios (the corresponding sides are in proportion).	FM2: 153-155

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	4. write and solve proportions to measure distances indirectly.	FM2: 155, 156 158-161
	5. write and solve proportions to determine the dimensions of a scale drawing.	FM1: 52-57
C. Finding Square Roots; the Pythagorean Rule	1. make a list of squares of whole numbers from 1 to 15.	
	2. state that a perfect square can be written as the product of two equal factors.	
	3. state that the square root of a number is one of its two equal factors.	
	4. find the square roots of various perfect squares by trial and error (include fractions and decimals).	
	5. state that the hypotenuse is the longest side of a right triangle and it is opposite the right angle.	
	6. state that the legs of a right triangle form the right angle.	FM2: 162-163
	7. locate the hypotenuse and legs in given right triangles.	
	8. state that a and b in the Pythagorean Rule ($a^2 + b^2 = c^2$) are the legs, and c is the hypotenuse.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	9. use the Pythagorean Rule and the concept of finding a square root (by trial and error) to find the hypotenuses of several right triangles with whole numbered sides. (Use only Pythagorean triples.)	FM2: 165, 166-168
	10. calculate the length of the leg of a right triangle, using the Pythagorean Rule.	FM2: 168
	11. classify a triangle as a right triangle using the converse of the Pythagorean Rule.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
IX. GRAPHICAL INTERPRETATION		
A. The Value of Graphs	1. provide examples in which collecting data is necessary (e.g., insurance, weather forecasting, consumers' needs).	FM1: 58
	2. describe ways of organizing data: lists, charts, graphs.	FM1: 58
	3. identify various types of graphs found in periodicals: pictograph, bar graph, line graph, circle graph.	FM1: 58, 59
B. Reading and Interpreting Pictographs	1. label a given graph with an appropriate title.	FM1: 60
	2. read the scale on a pictograph.	FM1: 60, 61, 62, 63
	3. read cut symbols on a pictograph as smaller amounts or fractions.	FM1: 60, 61, 62, 63
	4. translate the symbols on a pictograph into numerical quantities.	FM1: 60, 61, 62, 63
	5. interpret (extrapolate new information or form opinions) using the data on the pictograph.	FM1: 60, 61 62
C. Reading and Interpreting Bar Graphs	1. identify bar graphs as a type of graph that can illustrate data more accurately than pictographs.	FM1: 64
	2. translate the scale units used on a bar graph into numerical quantities.	
	3. read and interpret horizontal and vertical bar graphs.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	4. read and interpret bar graphs with double sets of data from various sources.	FM1: 66
D. Reading and Interpreting Line Graphs	1. identify line graphs as a type of graph that illustrates trends more effectively than bar graphs.	
	2. state that bar graphs illustrate discrete data while line graphs show a continuous series, process or tendency.	
	3. read and interpret line graphs from many sources.	FM1: 68
	4. identify situations in which the choice of scales for line graphs and bar graphs can distort the facts being shown, and even mislead the reader.	FM1: 70
E. Reading and Interpreting Circle Graphs	1. state that the sum of all the parts of a quantity is 100%.	
	2. identify circle graphs as a good graph form to use when illustrating the subdivisions of a whole quantity (100%).	
	3. express simple fractions as equivalent percents.	
	4. find a percent of a number.	FM1: 72
	5. read and interpret circle graphs.	FM1: 71, 72
F. Summarizing Graphs	1. select the most appropriate type of graph to display a given set of data for a specified purpose.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
X. COORDINATE GRAPHS: LOCATING POINTS		
	1. describe a seat location in the classroom using two numbers (coordinates): row and seat number in the row.	
	2. distinguish between location (3, 2) and location (2, 3), and define an ordered number pair.	FM1: 176
	3. state the coordinates of a point on a grid which corresponds to the seats in the classroom (containing only positive horizontal and vertical units -- quadrant I).	
	4. identify coordinates of a point to the left of 0 on the horizontal number line as negative, and similarly, coordinates below 0 on the vertical number line as negative.	FM1: 174, 175
	5. label the horizontal number line with x, and call it the x-axis.	FM1: 175
	6. label the vertical number line with y, and call it the y-axis.	FM1: 175
	7. locate the origin and state its coordinates.	FM1: 175
	8. read and locate the coordinates of given points on the coordinate plane, including points on the axes.	FM1: 177-181
	9. plot points on the coordinate plane, given their coordinates.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XI. STATISTICS		
A. Introduction to Statistics	1. provide examples in which accurate numerical analysis of data is needed to make decisions in many fields: retailing, science, politics.	FM1: 75
	2. define statistics as the science of collecting, organizing, and interpreting data.	FM1: 75
B. Organizing and Interpreting Data Using a Frequency Table	1. read a set of random data and set up a system of classes (categories for the purpose of tallying).	FM1: 75
	2. tally the data according to the scheme determined above.	FM1: 78, 79
	3. define frequency.	
	4. organize the data by completing a frequency table.	FM1: 79
C. Defining and Determining the Mode	1. define mode.	FM1: 81
	2. determine the mode from frequency tables, including the Class Survey Data.	FM1: 81, 82
D. Defining and Determining the Mean	1. define mean as the sum of the values divided by the number of values (average).	FM1: 83
	2. apply the definition to find the mean, given a set of data.	FM1: 83, 84
E. Defining and Determining the Median	1. arrange a set of data in numerical order.	

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CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. define median for an odd number of data and for an even number of data.	FM1: 85, 86
	3. apply the definitions to find the medians in given sets of data.	FM1: 86, 87

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XII. PROBABILITY		
A. Understanding the Concept of Probability	1. cite uses of chance or probability.	
	2. guess (or predict) the outcomes of an experiment with a spinner, dice, or coins.	FM1: 88
	3. perform the experiment, tally the outcomes, and compare them to the predictions.	FM1: 89, 90
	4. express a probability as the ratio of two quantities: $\frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$	FM1: 99
	5. state the relationship between the number of outcomes, the number of successes, and the number of failures.	FM1: 91, 92
	6. use the rule that the probability of success plus the probability of failure = 1, to calculate the probability of an event given the probability of its complement.	FM1: 92, 93, 94
B. Understanding the Concept of Sample Space	1. list all the possible outcomes of a probability experiment (the sample space).	FM1: 95, 96
	2. determine the probability of a given outcome in an experiment after listing the sample space.	FM1: 96, 98, 99
C. Predicting Outcomes Based Upon the Probability Ratio	1. describe a sample space and determine the probability of a specified outcome before performing the experiment.	FM1: 100

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. calculate predicted numbers of outcomes by using the formula: Predicted number of successful outcomes to an experiment = probability of success times the number of trials.	FM1: 100
XII. CONSUMER-RELATED MATHEMATICS		
A. Making Change	1. express the amount of money in a $1, $5, $10 or $20 bill in terms of dollars and cents ($5 = 5.00).	
	2. subtract a total cost from the amount of money in a $1, $5, $10 or $20 bill to determine the change returned.	
B. Unit Pricing		
-Understanding unit pricing and using unit pricing labels	1. express separate monetary value written in cents as a decimal number (67¢ = .67).	
	2. state the number of ounces, pints, and quarts in one gallon.	
	3. calculate and compare the unit prices of goods sold by the pint, quart, or gallon, using the price per gallon as the unit price (only whole number multiples of quantities are required).	FM1: 102
	4. read and compare samples of supermarket unit price shelf markers to find the best buy.	FM1: 103
-Computing the actual price given the unit price	1. find the product of two decimal numbers, and round it off to the nearest hundredth.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. round off a monetary amount to the nearest cent.	
	3. apply the formula: actual price = unit price times weight (or quantity) to compute the actual price of an item.	FM1: 105, 113
-Computing the unit price, given the actual price	1. find the quotient of a decimal number (monetary value) by a whole number, expressed to the nearest cent.	
	2. find the quotient of one decimal number by another to the nearest hundredth.	
	3. apply the formula: unit price = actual price divided by the weight (or quantity) to compute the unit price of an item.	FM1: 107, 108 113
-Computing the weight (or quantity), given the actual price and the unit price	1. apply the formula: weight (or quantity) = actual price divided by unit price, to determine the weight of an item.	FM1: 110-112, 113
C. Earning Money		
-Computing a salary based on an hourly wage	1. define gross wages.	FM1: 114
	2. complete a payroll table which does not include any overtime pay.	FM1: 114
	3. determine the number of hours between two given clock times, including A.M. and P.M. times.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
-Computing "time and a half" overtime wages	1. find one half of any dollar amount to the nearest cent.	
	2. find 1½ times any dollar amount to the nearest cent.	
	3. determine a worker's overtime hours.	
	4. complete a payroll table which includes time and a half overtime pay.	FM1: 116
-Computing salary based on piece work	1. state the relationships between weeks, months and one year.	
	2. compute the annual salary, given salary for one week or one month.	FM1 121
	3. compute the weekly or monthly salary, given the annual salary.	FM1: 121
	4. compare the advantages of being paid for a given job by the hour, week, month or year.	FM1: 120
-Computing salary based on commission	1. convert a whole number percent between 1% and 99% to a decimal number.	
	2. compute a percent of a number by finding a product.	
	3. calculate a commission.	FM1: 122
	4. complete a payroll table of wages based on commissions.	FM1: 123

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
D. Calculating Interest		
- Simple interest	1. use the words deposit, withdrawal, principal, interest, interest rate and balance to describe banking procedures.	
	2. explain why a bank pays interest.	FM2: 43
	3. identify the principal, rate and time period in a bank interest problem.	
	4. state that interest rates are always expressed as an annual rate.	
	5. find the product of three numbers where only one or two of them are whole numbers and the others are decimal fractions.	
	6. compute interest using the simple interest formula I = prt, where t is an integral number of years.	FM2: 44, 45
- Simple interest or loan interest for periods of less than one year	1. reduce fractions with denominators of 12 to lowest terms.	
	2. convert mixed numbers to improper fractions in lowest terms.	
	3. express any number of months as a "fraction of a year," including examples of more than 12 months.	FM2: 46
	4. express a whole number percent between 1% and 99% as a fraction in lowest terms.	

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CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	5. express a rational percent with a denominator of 2 or 4 as a fraction in lowest terms (6 1/2%, 7 3/4%, etc.)	
	6. find the product of three rational numbers with reducing.	
	7. calculate interest on a loan or bank interest using I = prt for any period expressed as an integral number of months.	FM2: 46, 47
- Compound interest	1. state that when calculating simple interest:	
	a. the interest is figured all at once for the entire period,	
	b. the principal does not change during the period.	
	2. state that when calculating compound interest:	
	a. the interest is figured separately for each unit of time period (e.g., per year for annual compounding),	
	b. the principal increases after each unit of the time period (e.g., this year's principal equals last year's principal plus last year's interest).	
	3. calculate interest compounded annually.	FM2: 50, 51

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
- Repaying a loan in equal installments	1. calculate the interest amount charged on a loan.	FM2: 52
	2. calculate the total amount borrowed (interest plus "net proceeds").	
	3. state that each installment is the quotient of the total amount borrowed divided by the number of equal installments.	FM2: 52
	4. compute each installment on a loan.	FM2: 52, 53
- Understanding "bank discount"	1. compute the interest amount charged on a loan.	
	2. compute the actual amount of money available to the borrower (net proceeds).	
	3. compute the amount that must be borrowed in order to have a given amount of net proceeds.	FM2: 54, 55
E. Calculating the Selling Price When Discounting	1. calculate a fraction of a number (with reducing and no remainder).	
	2. compute the amount of a discount if the discount rate is a unit fraction.	FM2: 57
	3. compute the selling price (net price) if the discount rate is a unit fraction.	FM2: 57, 58
	4. convert several common percents to their fractional equivalents (10%, 20%, 25%, 33 1/3%, etc.)	
	5. compute the net price where the discount rate is expressed as a percent (which is equivalent to 1/2, 1/3, 1/4, etc.)	FM2: 60-62

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
F. Calculating Sales Tax	1. calculate the sales tax by finding the product of selling price and sales tax rate.	FM2: 63-65
	2. read a sales tax table and use it to determine sales tax.	
G. Calculating a Purchase Price Requiring Discounting and Sales Tax	1. state that sales tax is added after the net price is calculated.	FM2: 66
	2. calculate the purchase price for goods advertised in a newspaper sale, including discounting and sales tax.	FM2: 67
H. Double (Successive) Discounts	1. calculate "double" discounts by finding the net price after the first discount, and then finding the net price after the second discount.	FM2: 69
	2. state that "double" discounts cannot be found by adding the two discount rates.	
	3. compute the discount rate equivalent to the result of a discount of n% and p% by finding the product of (100 - n)(100 - p)%.	FM2: 70
	4. calculate double discounts using the equivalent rate.	FM2: 70
I. "Taxicab" Arithmetic	1. provide examples of "step-function" price scales from real life (e.g., taxi rates, postage rates, phone rates, etc.)	
	2. determine the number of units to be charged where the given number of units is not a whole number (postage on 13¼ ounces will be charged as 14 ounces).	
	3. compute the total price by adding the initial charge to the product of the unit rate by the number of additional units.	RCT exam 6/80, #58

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XIV. METRIC MEASUREMENT		
A. Linear Measurement	1. identify the meter as the basic unit of linear measure in the metric system.	
	2. express the numerical relationships between one millimeter, one centimeter and one kilometer and the meter.	FM1: 124
	3. identify (visually) the meter as the metric unit closest in size to the yard.	
	4. arrange the units millimeter, centimeter, meter and kilometer in order of size.	
	5. provide examples of appropriate uses for the millimeter, centimeter, meter and kilometer.	FM1: 125
	6. write a fraction with a denomenator of 10, 100 or 1000 as a decimal number.	
	7. use the fact that 1 m = 100 cm in order to express a given multiple of 10 cm as a fraction of a meter (e.g., 80 cm = .8 m, 140 cm = 1.4 m).	
	8. apply the rules of converting meters to centimeters (multiply by 100) and centimeters to meters (divide by 100).	
	9. measure lengths in centimeters and meters with a ruler.	FM1: 128
	10. recognize and read the millimeter markings on a ruler.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	11. convert measurements from millimeters to centimeters and vice versa.	FM1: 129
	12. measure lengths in centimeters.	FM1: 130
	13. state that a kilometer is a little more than half a mile.	
	14. convert measurements from meters to kilometers and vice versa.	FM1: 132
	15. compute distances, fuel costs, etc. in problems using metric units.	FM1: 133
B. Units of Weight	1. identify the gram as the basic unit of weight in the metric system.	FM1: 135
	2. state that there are about 30 grams to one ounce (no conversion of units is to be made).	
	3. express the relationships between one gram, one milligram and one kilogram.	FM1: 135
	4. provide examples of appropriate uses for the milligram, gram and kilogram.	FM1: 136
	5. convert grams to kilograms and vice versa.	FM1: 137
	6. read a metric weighing scale.	FM1: 137, 138
	7. convert grams to milligrams and vice versa.	FM1: 139
	8. state the relationship between the kilogram and the metric ton.	FM1: 141
	9. convert from kilograms to metric tons and vice versa.	FM1: 142, 143

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
C. Units of Temperature	1. state the names of the two common temperature scales.	
	2. recognize certain significant temperatures on the Celsius scale: freezing point of water, boiling point, normal body temperature, etc.	
	3. convert from one scale to the other by reading a chart showing parallel temperature scales.	FM1: 144
D. Liquid Measure and Volume	1. identify the liter as the basic unit of liquid capacity (or volume) in the metric system.	FM2: 125
	2. identify the liter as the metric unit closest in size to a quart.	FM2: 125
	3. express the relationships between one milliliter, one liter, and one kiloliter.	FM2: 125
	4 arrange the units milliliter, liter, and kiloliter in order of size.	
	5 provide examples of appropriate uses for the milliliter, liter, and kiloliter.	FM2: 125, 126
	6. convert liters to milliliters, and kiloliters to liters, and vice versa.	FM2: 128, 129
	7. solve a variety of problems involving metric liquid measures.	FM2: 129, 130 131, 132, 133
	8. state that one milliliter of liquid measure is equivalent to one cubic centimeter of volume.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	9. find the volume of a rectangular prism ($V = lwh$) using metric units.	
	10. express volume in cubic centimeters and liters in various problem situations.	FM2: 135

Algebra

WORK STUDY SKILLS: ALGEBRA

FOLLOWING DIRECTIONS

Taking notes from oral directions requiring several steps.

Applying a pattern for following written directions:

- Reading for overview
- Rereading for specific steps
- Carrying out steps
- Checking for correct completion

Applying skills in following directions to the organization of homework, classwork and examination papers (e.g., the display of work, the location of final answers, the calculation of answers to correct degrees of accuracy, etc.)

Translating directions concerning the rounding-off of answers into the expression of answers with the correct number of decimal places.

Working in small groups to solve a problem or investigate a relationship.

LOCATING INFORMATION

Interpreting and rephrasing questions.

Using the table of values of trigonometric ratios to locate the values for a specific angle, or, given the value of the ratio, to locate a specific angle.

Using a table of square roots to locate the square root or square of a given number.

Referring to notes for previous definitions, formulas, techniques.

Finding the numerical information expressed in verbal problems.

RECALLING INFORMATION

Stating previous formulas, definitions and techniques.

Using mnemonic devices for remembering relationships (e.g., SOHCAHTOA).

Remembering approximate values of pi.

Remembering arithmetic algorithms for long division to aid in the development of algorithms for the division of polynomials and computing square roots.

Remembering the correct order of operations.

INTERPRETING INFORMATION

Choosing items of given information, from the statement of a verbal problem, that would be needed for finding a solution.

Determining which property of the operations or of equality is illustrated by a particular algebraic sentence.

Writing algebraic phrases and sentences which translate the relationships between quantities expressed verbally.

Expressing the product of a number of identical factors by using an exponent.

Using the radical sign to indicate roots.

Reading numerical information from given diagrams on the number line or the coordinate plane.

Drawing a coordinate graph from given numerical information about the coordinates of points or given equations and inequalities.

Reading a table of values for a variable or variables and translating the information in it to a graph.

Providing examples from real life of business, consumer or geometric situations illustrating specific relationships between quantities.

ORGANIZING INFORMATION

Refining skills in note taking.

Maintaining lists of definitions, formulas and techniques.

Displaying work associated with the solution of numerical and algebraic problems.

Making sketches based upon the conditions in verbal problems about geometric figures.

Preparing a table of values to aid in the graphing of the solution set of an equation or inequality.

Preparing a chart or other labeling scheme for the arrangement of information extracted from verbal problems.

MANIPULATING PHYSICAL MATERIALS

Measuring line segments with a ruler (English and metric).

Measuring an angle with a protractor.

Using colored pencils, shadings, dotted lines, etc., to highlight the details of a graph.

Using graph paper in coordinate geometry exercises.

Using a straightedge to draw line segments on coordinate graphs, number lines, geometric diagrams, tables of values, etc.

MINIMUM ESSENTIALS: ALGEBRA

There are two sets of references to curriculum bulletins given for these performance objectives. References preceded by the code M9 are found in Mathematics Ninth Year: 1966-67 Series Curriculum Bulletin. References preceded by the code X9 are found in Mathematics 9, Term 1 or Term 2, experimental bulletins produced under project number 6013.

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
I. FUNDAMENTAL CONCEPTS	1. recognize the sets of whole numbers, natural numbers, and positive rational numbers.	X9-1: 5, 6
	2. find the sum , difference, product, and quotient of any whole numbers, decimal numbers, or fractions.	M9: 2, 3
	3. write equations to illustrate the commutative and and associative properties for addition and multiplication.	X9-1: 47
	4. write an equation to illustrate the distributive property of multiplication over addition or subtraction.	X9-1: 41, 99
	5. express the relationship between two numerical expressions using $<$, $>$, $=$, $\neq$.	M9: 4
	6. specify the base and the exponent of a power (e.g., 6^3).	M9: 52, 53
	7. use the terms "square" and "cube" when referring to the second and third powers of numbers.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

8. find powers of any rational number.

9. follow the rules of the order of operations to simplify numerical expressions (including exponents and grouping symbols). M9: 4-7, 53, 54, 55 X9-1: 12-15

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
II. TRANSLATION OF PHRASES AND SENTENCES	1. state the mathematical operation which is indicated by such words as sum, difference, etc.	M9: 8 — X9-1: 10, 11
	2. translate phrases that refer to a grouping of terms.	M9: 8 — X9-1: 10, 11, 16-17
	3. express a numerical phrase verbally.	X9-1: 11, 17

CONTENT AREA AND TOPIC		PERFORMANCE OBJECTIVES	REFERENCES
		The student will be able to ...	
III. SIGNED NUMBERS	1.	locate any signed number on a number line.	X9-1: 6, 7, 8
A. Addition	2.	state that the additive identity is zero.	M9: 66 / X9-1: 65
	3.	find the additive inverse of any signed number.	M9: 9, 66, 70 / X9-1: 61
	4.	relate the positions of two numbers on the number line to their order (e.g., $-8 < -6$ because -8 is to the left of -6).	X9-1: 62
	5.	state the dual interpretations of "+" and "-" as signs of operation and direction.	X9-1: 61
	6.	evaluate expressions involving absolute value and signed numbers (e.g., $\|-5\|$, $\|8-2\|$, $4 + \|-31\|$).	X9-1: 63, 64, 66
	7.	state and apply the rules for addition of signed numbers (using integers, decimals, and rationals) to two, three, or four numbers, written horizontally or vertically.	X9-1: 67-69
	8.	determine whether or not a set of signed numbers is closed for addition.	X9-1: 60
	9.	write equations to illustrate the commutative and associative properties for addition of signed numbers.	M9: 10-16 / X9-1: 65, 69
B. Multiplication	1.	state the rules for multiplying two signed numbers.	
	2.	state a rule for determining the sign of a product of more than two signed numbers.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES	
	The student will be able to ...		
	3. apply the rules to find the product of two or more signed numbers.	M9: 21	X9-1: 70-75
	4. apply the rules of multiplication to simplifying expressions with exponents [e.g., $(-3)^3$, $(5-7)^3$].		
	5. write equations to demonstrate the associative and commutative properties of multiplication of signed numbers.	M9: 21-25	X9-1: 12
	6. write an equation to illustrate the distributive property of multiplication over addition for signed numbers.	M9: 25-29	
	7. determine whether or not a set of integers is closed for multiplication.		X9-1: 73
	8. state that the multiplicative identity is 1.		X9-1: 72
C. Subtraction	1. read -(-a) as "the opposite of negative a," where a is any rational number.	M9: 29	X9-1: 80, 83
	2. state that subtraction and addition are inverses.	M9: 34-35	X9-1: 80
	3. state the rule that subtraction of a number is equivalent to the addition of its opposite (additive inverse).		X9-1: 80
	4. apply the rule to find the difference of two signed numbers.	M9: 36, 37	X9-1: 80-81, 83-85

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	5. determine whether or not a set of integers is closed for subtraction.	
	6. write sentences to illustrate that the commutative and associative properties do not apply to subtraction.	M9: 37 — X9-1: 81
	7. write an equation to illustrate the distributive property of multiplication over subtraction.	M9: 39
	8. recognize the equivalence of expressions such as $6 - 9 = 6 + (-9)$ and $5 - (-3) = 5 + 3$.	M9: 36 — X9-1: 85
	9. simplify numerical expressions involving both sums and differences (e.g., $6 + (-2) - 7 - (-9) = ?$).	
D. Division	1. recognize that division and multiplication are inverses.	M9: 39 — X9-1: 76
	2. state that the multiplicative identity is 1.	M9: 66
	3. write the multiplicative inverse (reciprocal) of any non-zero rational number.	M9: 67, 68 — X9-1: 72
	4. write division problems expressed verbally or with the symbols $\div$ or $)$ using fraction notation.	
	5. state the rules for division of signed numbers.	
	6. apply the rules to find the quotient of two signed numbers.	M9: 40-41 — X9-1: 76-78

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

7. determine whether or not a set of integers is closed for division

8. write a sentence to illustrate that division is not commutative or associative. — M9: 46

9. rewrite expressions of the form $\frac{a + b}{n}$ as $\frac{1}{n}(a + b)$, where a, b and n are integers.

10. write an equation to illustrate the distributive property of division over addition and subtraction. — M9: 46-47 — X9-1: 78

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CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
IV. USING LETTERS AS VARIABLES IN ALGEBRA		
	1. distinguish between an open sentence and a phrase.	M9: 48-49 / X9-1: 23, 24
	2. distinguish between an open sentence and a sentence containing no variables.	X9-1: 23, 24
	3. write the solution set of an open sentence (including $<$, $=$, and $>$) given a replacement set, by trial and error.	M9: 49, 50, 51 / X9-1: 23-25
	4. evaluate formulas and algebraic expressions using a specified replacement set (including area, circumference, and perimeter formulas).	M9: 50, 51, 54-57 / X9-1: 19-22, 78, 84
	5. translate English phrases and sentences into algebraic expressions and open sentences (including equations and inequalities).	M9: 57-60 / X9-1: 26-29
	6. recognize illustrations of the associative, commutative, distributive, and inverse operation properties when they are presented as algebraic sentences.	M9: 61-75

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
V. OPEN SENTENCES IN ONE VARIABLE		
A. Solving Equations	1. solve equations of the form $x + a = b$, $x - a = b$, $ax = b$ and $\frac{x}{a} = b$ by using inverse operations (including equations with the variable in the right member, such as $b = x + a$).	M9: 34 — X9-1: 30-35, 87
	2. solve equations of the form $\frac{ax}{b} = c$ by using inverse operations.	X9-1: 40
	3. solve equations of the form $ax + b = c$ and $\frac{x}{a} \pm b = c$ by using inverse operations to change them into simpler equivalent equations.	M9: 76-77 — X9-1: 39-40 87, 88
	4. use the distributive property to simplify expressions of the form $a(bx + c)$.	
	5. use the distributive property to simplify expressions of the form $ax + bx$.	X9-1: 86
	6. solve equations of the form $a(bx + c) = d$ and $ax + bx + c = d$.	M9: 77, 78, 79 — X9-1: 88, 90-92
	7. solve equations of the form $ax = bx + c$ and $ax + b = cx + d$.	M9: 115-117
	8. check the solutions to all of the above equations.	
	9. solve a literal equation for a specified variable.	X9-1: 149-151

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
B. Solving Verbal Problems	1. represent the unknown quantities in verbal problems as letter variable or algebraic expressions.	M9: 79 — X9-1: 50-55
	2. translate the conditions of a verbal problem into an equation.	
	3. solve the equation and use the solution to write the answer(s) to the problem.	
	4. solve:	
	a. simple number problems	M9: 79-80 — X9-1: 50-51, 56
	b. consecutive integer problems	M9: 80-81 — X9-1: 93-95
	c. percentage problems	X9-1: 36-38
	d. geometry-related problems (perimeter, sum of the measures of the angles of a triangle, complementary and supplementary angles)	M9: 85, 86 — X9-1: 51, 52-54, 56-58
	e. coin problems	M9: 81-83 — X9-1: 143-147
	f. motion (rate - time - distance) problems	M9: 87-90 — X9-1: 152-159
	g. investment problems	M9: 84
	h. mixture problems	M9: 90-91
	5. eliminate irrelevant answers by recognizing the appropriate replacement set for the variable.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	6. check solutions in the problem conditions.	
C. Solving and Graphing Inequalities in One Variable on the Number Line	1. write the solution set to an inequality ($<$, $\leq$, $>$, and $\geq$) given a replacement set.	M9: 124-125 X9-1: 100
	2. describe the solution set of an inequality as infinite.	X9-1: 100-101
	3. graph the solution set of an inequality of the form $x < a$, $a < x$, etc. on a number line.	M9: 126 X9-1: 100-102
	4. solve and check an inequality of the form $ax < b$, where a is negative.	M9: 131 X9-1: 104
	5. solve, graph, and check the solution sets of inequalities of the form $ax + b < c$, $ax + b < cx + d$, $a(bx + c) < d$, etc.	M9: 128-134 X9-1: 103
	6. interpret, solve, and graph the solution sets of combined inequalities such as $a \leq x \leq b$.	M9: 127 X9-1: 105-106
	7. write and interpret expressions written in "set-builder" notation.	X9-1: 106-107
	8. graph the solution sets of inequalities expressed in "set-builder" notation.	

CONTENT AREA AND TOPIC		PERFORMANCE OBJECTIVES	REFERENCES	
		The student will be able to ...		
VI. POLYNOMIALS				
A. Coefficients, Factors and Exponents	1.	express any composite number as a product of factors.	M9: 51	
	2.	demonstrate the difference between x^2 and $2x$ by evaluating expressions using various replacements for x.	M9: 52	
	3.	identify the coefficient, exponent, and base in any expression such as $4y^7$.	M9: 53	X9-1: 110
B. Fundamental Concepts	1.	state how many terms a given polynomial has.	M9: 93	
	2.	categorize a polynomial as a monomial, binomial, or trinomial.	M9: 94, 95, 96	X9-1:111, 114
	3.	state the degree of a given polynomial.	M9: 94	X9-1: 113
	4.	state that x is equivalent to $1x^1$.		
	5.	state the coefficient of any term, including negatives (e.g., the x-term in $5x^2 - 3x + 2$).	M9: 94	X9-1: 110
	6.	write a polynomial in descending order.	M9: 95	X9-1: 113
C. Addition	1.	use the distributive property to add similar (like) terms.	M9: 96-97	X9-1: 109-111
	2.	add polynomials in horizontal or vertical formats.	M9: 97-99	X9-1: 115-118

CONTENT AREA AND TOPIC		PERFORMANCE OBJECTIVES	REFERENCES	
		The student will be able to ...		
D. Subtraction	1.	write the additive inverse of any monomial.		X9-1: 110, 111
	2.	write the additive inverse of any polynomial.	M9: 109-111	X9-1: 119
	3.	show that the sum of a polynomial and its additive inverse is zero.	M9: 109-111	
	4.	state that subtraction is the same as adding an additive inverse (opposite).	M9: 108-109	X9-1: 119
	5.	subtract one polynomial from another when written in a horizontal or vertical format.	M9: 111-113	X9-1: 120
	6.	solve and check equations that include addition or subtraction of polynomials.	M9: 114	
E. Multiplication of a Monomial by a Monomial	1.	write a power such as x^5 as the product of its factors.		X9-1: 121
	2.	apply the rule $x^a x^b = x^{a+b}$, where a and b are whole numbers.	M9: 100	X9-1: 121-122
	3.	demonstrate the difference between $(3x)^2$ and $3x^2$ by using various replacements for x.		X9-1: 123
	4.	find products of two or more monomials.	M9: 101	X9-1: 122-123
	5.	find powers of monomials.		X9-1: 122-123
F. Multiplication of a Polynomial by a Monomial	1.	apply the distributive property to find the product of a polynomial by a monomial.	M9: 101-102	X9-1: 124-125

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. multiply a polynomial by a monomial to express the area of a rectangle algebraically.	M9: 101, 102
	3. solve and check equations using the product of a polynomial by a monomial.	M9: 103 X9-1: 125
	4. state that the product of a polynomial and -1 is the additive inverse of the polynomial.	
G. Multiplication of a Binomial by a Binomial	1. use the FOIL method to assist in distributing the multiplication in the horizontal format.	M9: 104, 107 X9-1: 127
	2. multiply two binomials in the vertical format.	M9: 105, 106, 108 X9-1: 126
	3. apply the rule $(a + b)(a - b) = a^2 - b^2$ to calculate the product of conjugate binomials.	
	4. expand a power of a binomial $(x + a)^n$ by multiplying, where n = 2 or 3.	X9-1: 127
	5. solve verbal area problems involving the multiplication of binomials (requiring solution of only linear equations).	
H. Division of a Monomial by a Monomial	1. state that the quotient of any non-zero quantity divided by itself is one.	M9: 214, 215 X9-1: 130, 131
	2. state the rule $x^a \div x^b = x^{a-b}$, where a and b are whole numbers, and $a > b$.	M9: 175-177 X9-1: 130

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	3. find the quotient of two monomials where division is expressed in the fraction form, or with the symbols $\div$ or $\overline{)}$.	X9-1: 131-133
	4. check the division by multiplying.	M9: 176
I. Division of a Polynomial by a Monomial	1. rewrite expressions of the form $\frac{ax + b}{n}$ as $\frac{1}{n}(ax + b)$.	
	2. apply the distributive property to find the quotient of a polynomial by a monomial.	M9: 177-179 X9-1: 134-136
	3. check the division by multiplying.	
J. Division of a Polynomial by a Binomial	1. write a polynomial in descending order, and fill in "missing" terms with zeros.	M9: 184 X9-1: 139
	2. use the process of arithmetic long division as a guide to divide a polynomial by a binomial (resulting in a quotient with remainder zero).	M9: 180-181 X9-1: 137-139
	3. divide a polynomial by a binomial and express the remainder as a fraction.	M9: 182, 184 X9-1: 140-142

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

VII. LINEAR EQUATIONS AND INEQUALITIES IN TWO VARIABLES

A. Plotting Points and Graphing One Equation in Two Variables

1.	provide solutions to an equation in two variables by trial and error.	
2.	state that each solution to an equation in two variables includes two numbers.	M9: 135, 136 X9-1: 166
3.	demonstrate that (a, b) and (b, a) are different by substituting them into an equation (e.g., (6, 4) and (4, 6) do not both satisfy the equation $2x + y = 16$).	M9: 135
4.	label the origin, the x-axis, the y-axis and the quadrants on the coordinate plane.	
5.	locate points on the coordinate plane, given their coordinates.	M9: 139-146 X9-1: 162, 164
6.	identify the abscissa and the ordinate of any point on the coordinate plane (including the origin and other points on the axes).	X9-1: 162, 163
7.	determine whether a given number pair is a solution to an equation in two variables or not.	M9: 138, 150
8.	state that the solution set of $ax + by = c$ is infinite.	M9: 137

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

9. solve an equation of the form ax + y = b for y in terms of x.	M9: 147	
10. find three or four solutions to the equation ax + y = b, write them as number pairs and plot them.		X9-1: 166-167
11. state that the graph of an open sentence of the form ax + by = c is always a line and that the equation is called a linear equation.		X9-1: 167
12. state that the line is the set of all points and only those points whose coordinates satisfy the equation.	M9: 148	X9-1: 167
13. solve equations for y in terms of x, such as ax + by = c where a is a multiple of b, or where a and b are relatively prime and ax - y = b.		X9-1: 169
14. find the second coordinate of a point given the equation of a line through the point and one coordinate of the point.		X9-1: 169
15. find three or four solutions to equations in various forms (in two variables) and plot the graphs of the solution sets, using tables to organize the solutions.	M9: 148-149	X9-1: 168-169
16. state that a minimum of two points is needed to graph a line, but a third point is used as a check.		X9-1: 168

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	17. draw the graphs of equations such as $x = c$, $y = c$, $y = \|c\|$, and $y = \|x\|$.	M9: 149, 151 X9-1: 173-174, 176-177
	18. determine the x- and y-intercepts algebraically for an equation of the form $ax + by = c$, where a and b are non-zero integers.	X9-1: 171-173
B. Graphing By the Slope-Intercept Method	1. describe the direction of a line on a graph in terms of the change in y and the change in x from one point on the line to another point to its right.	X9-1: 178-179
	2. express the slope of a line as a ratio, given its graph, and show that the slope is constant.	X9-1: 179-181
	3. determine whether the slope of a line is negative, zero, or positive, or that the line has no slope, by looking at the graph of the line.	X9-1: 182
	4. determine the y-intercept of a line from its graph.	X9-1: 185
	5. determine the slope and y-intercept of a line from its equation in the form $y = mx + b$.	X9-1: 183-185
	6. rewrite equations in the form $y = mx + b$.	
	7. plot the graph of an equation of the form $y = mx + b$ and $ax + by = c$ using the slope and y-intercept method.	X9-1: 187-189
	8. determine whether two lines with equal slopes are parallel or not .	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
C. Solving a System of Equations Graphically	1. state that one equation in two variables has an infinite number of solutions, but a system of two such equations can have exactly one common solution which satisfies both equations simultaneously.	M9: 156-157 X9-1: 194
	2. determine whether the graphs of two equations in two variables are parallel (inconsistent system), are the same line (dependent system), or intersect in exactly one point (consistent system).	M9: 158-162 X9-1: 198-200
	3. find the common solution of a consistent system of two equations in two variables graphically.	M9: 163 X9-1: 195-196
	4. check the common solution in both equations.	X9-1: 195
D. Solving a System of Equations Algebraically		
- Substitution method	1. state that one variable must be eliminated to solve a system of two equations in two variables.	X9-1: 211-212
	2. solve one of the equations for y in terms of x, or for x in terms of y, if necessary.	M9: 165
	3. substitute the expression for the appropriate variable in the other equation to form one equation in one variable.	M9: 165 X9-1: 211
	4. solve and check the solutions of a system of equations.	M9: 166, 167 X9-1: 206

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
- Addition method	1. form a third equation by finding the sum of the two given equations and demonstrating that all three equations have the same common solutions.	M9: 168-169 X9-1: 206
	2. state that the sum of two equations in two variables will have only one variable if the x-terms or y-terms in the original equations are additive inverses.	M9: 170 X9-1: 206
	3. solve systems of equations where the x-terms or the y-terms are additive inverses.	
	4. determine the appropriate multipliers for each equation in a system to produce a pair of equivalent equations in which the x-terms or the y-terms are additive inverses (where the equations are written in the form ax + by = c).	M9: 171 X9-1: 208-209
	5. solve and check systems of equations by addition, where multipliers must be used.	M9: 171-172 X9-1: 209-210
	6. select the most convenient method of solution for a given system of equations.	X9-1: 212
E. Solving Verbal Problems Using Two Variables	1. represent the two unknown quantities in terms of two variables according to the conditions of the problem.	X9-2: 16
	2. write two equations relating the two variables.	X9-2: 16, 17
	3. solve the system of equations algebraically and check the solution in the problem conditions.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	4. solve:	
	a. number problems	M9: 172
	b. business problems	X9-2: 17
	c. investment problems	X9-2: 23-27
	d. "dry" mixture problems	X9-2: 18-22
	e. digit problems	X9-2: 28-30
F. Graphing Inequalities		
- Linear inequalities in two variables	1. determine whether a given ordered number pair is a solution of an inequality (using $<$, $>$, $\leq$, and $\geq$).	M9: 138, 139). X9-1: 191-193
	2. graph the solution set of an equation such as $y = x + 1$ and indicate where solutions to $y < x + 1$ and $y > x + 1$ are found on the graph.	M9: 152-153, 154, 155 X9-1: 192-193
	3. state that the graph of an equation such as $y = x + 1$ divides the plane into two half-planes (regions) identified by $y < x + 1$ and $y > x + 1$.	M9: 152 X9-1: 192
	4. choose either a solid line or a broken line for the "edge" or boundary of the region determined by an inequality.	M9: 154 X9-1: 192

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	5. use one point in either half-plane to determine which half-plane is the solution set of the inequality.	M9: 155 X9-1: 192
	6. graph inequalities such as $y < c$, $x \leq c$, $y < ax + b$ and $ax + by \leq c$ by shading the appropriate region.	M9: 155-156
- Systems of linear inequalities	1. graph two linear inequalities in two variables and indicate the region (if any) where the common solutions are.	M9: 172-174 X9-1: 203
	2. show that points in the intersection of the solution sets satisfy both inequalities by testing one point in the intersection and one point outside of it in both inequalities.	X9-1: 203

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
VIII. FACTORING POLYNOMIALS AND SOLVING QUADRATIC EQUATIONS		
A. Greatest Common Monomial Factor (gcf)	1. identify prime numbers less than 100.	M9: 187
	2. write any composite number less than 100 as a product of primes.	M9: 187 X9-2: 49
	3. write all of the factors of a given whole number.	X9-2: 34
	4. find the greatest common factor (gcf) of any two whole numbers less than 100.	M9: 187-188
	5. express a monomial as the product of its prime factors.	
	6. write all of the factors of a given monomial.	X9-2: 34
	7. determine the gcf of two monomials.	M9: 188 X9-2: 34
	8. use the distributive property of multiplication over addition to express the product of a monomial and a binomial.	M9: 189 X9-2: 35
	9. determine the gcf of the terms of a binomial or trinomial and express the polynomial as the indicated product of the gcf and a polynomial (using the distributive property).	M9: 190 X9-2: 35
	10. check the factoring by multiplying.	X9-2: 35

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
B. The Difference of Two Squares	1. identify and recite the perfect squares from 1 to 144.	
	2. identify a rational or a decimal number as a pertect square (e.g., $\frac{1}{4}$, $\frac{4}{9}$, .04, .25).	
	3. define the square root of a perfect square as one of its two equal factors.	
	4. distinguish between monomials which are perfect squares and those which are not.	M9: 195
	5. write the square root of a perfect square monomial.	
	6. express the product of conjugate binomials as a binomial.	X9-2: 47
	7. state that the product of conjugate binomials is always expressed as the difference of squares of two monomials.	M9: 195 X9-2: 46-48
	8. distinguish between binomials which are a difference of squares and other binomials.	M9: 196
	9. use the pattern observed in the product of conjugate binomials to factor a difference of two squares.	M9: 196 X9-2: 47-48
	10. check the factoring by multiplication.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

C. Trinomial Factoring			
- Perfect square trinomials	1. find the product of two equal binomials : $(x + a)^2 = (x + a)(x + a) = x^2 + 2ax + a^2$.	M9: 191-192	X9-2: 37
	2. use the pattern of the coefficients to identify perfect square trinomials.	M9: 193	X9-2: 42
	3. express a perfect square trinomial as the product of its factors.	M9: 193	X9-2: 42
- Trinomials of the form $x^2 + bx + c$	1. multiply two binomials such as $(x + a)(x + b)$ by "inspection", as follows: a. state that a times b is the third term of the product (the constant). b. state that the sum of a and b is the coefficient of the x-term. c. recognize the relationships between the signs of the second terms of the binomials and the signs of the coefficients of the product.	M9: 197-198	X9-2: 36-37
	2. factor a trinomial of the form $x^2 + bx + c$.	M9: 198-200	X9-2: 40-41, 42-43
	3. state that if a trinomial of the form $x^2 + bx + c$ is factorable, it factors into two binomials in one and only one way.		

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

- Trinomials of the form $ax^2 + bx + c$, $a > 1$	1. find the product of binomials such as $(ax + b)(cx + d)$.	X9-2: 37
	2. state that a times c is the coefficient of the x^2-term.	M9: 201
	3. state that b times d is the third term of the product (the constant).	
	4. state that b plus d is NOT the coefficient of the x-term.	
	5. apply the rule that the coefficient of the x-term in the product $(ax + b)(cx + d)$ is the sum of the products of the first term of each binomial and the second term of the other binomial.	
	6. factor trinomials of the form $ax^2 + bx + c$ where a is 2 or 3.	M9: 201-202 X9-2: 44-45
D. Complete Factoring	1. state that a polynomial can have more than two factors.	X9-2: 49
	2. express a polynomial as the product of its gcf and another factor, and then continue factoring according to any other appropriate method.	M9: 202-203 X9-2: 49, 50
	3. state that if a polynomial has been factored completely, then it has been reduced to its prime factors.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	4. state that if a polynomial is factorable, then it can be factored completely in one and only one way.	
E. Solving Quadratic Equations	1. solve equations such as $3x = 0$, $x(x + 1) = 0$, $(x - 1)(x + 4) = 0$ by applying the rule: if $ab = 0$ then $a = 0$ or $b = 0$.	M9: 204-205, 206 X9-2: 51, 52-53
	2. determine the degree of an equation, and identify an equation as quadratic.	M9: 206, 263 X9-2: 31-32
	3. solve quadratic equations by factoring.	M9: 206-208 X9-2: 53, 54-57
	4. check both solutions of a quadratic equation.	
	5. state that a quadratic equation has two roots (which may be equal).	M9: 208 X9-2: 53
F. Verbal Problems Using Quadratic Equations	1. solve verbal problems which involve the solution of quadratic equations, including area and consecutive integer problems.	M9: 209-210 X9-2: 58-67
	2. check solutions to the verbal problems in the conditions of the problem.	
	3. identify those elements in the solution set of a quadratic equation which do not satisfy the conditions of the problem.	X9-2: 58-59, 62, 67

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

IX. FRACTIONS

A. Zero Denominators	1.	explain why division by zero and zero denominators result in expressions which are undefined.	X9-2: 69
	2.	determine the value of the variable in the denominator of a fraction for which the fraction is undefined.	M9: 212 X9-2: 69
B. Reducing (Simplifying) Fractions	1.	use the definition $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$ and the multiplicative identity to raise the terms of arithmetic fractions (e.g., $\frac{5}{7} \cdot \frac{4}{4} = \frac{20}{28}$).	M9: 213
	2.	state that the quotient of any number divided by itself (except zero) is one.	X9-2: 70
	3.	use the definition of multiplication of fractions and the multiplicative identity to simplify arithmetic fractions (e.g., $\frac{20}{25} = \frac{4 \cdot 5}{5 \cdot 5} = \frac{4}{5} \cdot 1 = \frac{4}{5}$)	M9: 213-214 X9-2: 70
	4.	find the gcf of two monomials.	X9-2: 70
	5.	reduce (simplify) fractions such as $\frac{x^2}{x^2}$, $\frac{2x+1}{2x+1}$, $\frac{-3x^2y}{-3x^2y}$ to one.	M9: 214 X9-2: 70, 73

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

	6. reduce fractions with monomial numerators and denominators using the gcf, the definition of multiplication of fractions and the multiplicative identity.	M9: 214, 215 X9-2: 71
	7. state that a fraction is in simplest form if there is no common factor in the numerator and denominator, other than one.	X9-2: 70
	8. reduce fractions where numerators and/or denominators are polynomials which must be factored.	M9: 216, 217 X9-2: 73-75
	9. show that a fraction such as $\frac{a + 4}{a}$ cannot be reduced, by replacing the variable with a number.	X9-2: 75
	10. check the result by replacing the variable with a number (avoid zero denominators).	
	11. state that $1 - x = -1(x - 1)$ and that $\frac{1 - x}{x - 1} = -1$.	M9: 216 X9-2: 76-77
	12. state that the quotient of additive inverses is -1.	
C. Multiplying Fractions	1. find the product of fractions where no reducing (simplifying) is involved.	M9: 217-218

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. find the product of fractions where factoring and reducing are involved (factor and reduce before multiplying).	M9: 219, 220 X9-2: 78-80
D. Dividing Fractions	1. divide rational numbers by applying the rule $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$ $(b, c, d \neq 0)$.	X9-2: 81-82
	2. write the reciprocal of an algebraic fraction.	M9: 220 X9-2: 82
	3. divide algebraic fractions and check, using numerical values for the variables (avoid zero denominators).	M9: 221-222 X9-2: 82
E. Adding and Subtracting Like Fractions	1. explain the addition of two fractions with like denominators using the distributive property: $\frac{a}{n} + \frac{b}{n} = a(\frac{1}{n}) + b(\frac{1}{n}) = \frac{1}{n}(a + b) = \frac{a + b}{n}$.	M9: 222-223
	2. add and subtract fractions with monomial numerators and like denominators.	M9: 223 X9-2: 84
	3. state that the lowest common denominator of two fractions is the same as the least common multiple of their denominators.	M9: 225 X9-2: 87
	4. determine the lowest common denominator of two or three fractions with unlike numerical denominators and monomial numerators.	M9: 225-227 X9-2: 88

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	5. combine two or three fractions with unlike numerical denominators and monomial numerators (including mixed expressions such as: $2 + \frac{3}{c}$).	M9: 227-229 X9-2: 88
	6. determine the least common multiple of two or three monomials.	X9-2: 90
	7. combine fractions having polynomial numerators and unlike monomial denominators.	X9-2: 89-90
	8. determine the least common multiple of two polynomials having common binomial factors, and having no common binomial factors.	
	9. combine fractions with unlike polynomial denominators.	M9: 229-230
	10. simplify the sum or difference of fractions with unlike polynomial denominators.	
G. Solving Equations With Fractions	1. solve equations of the form $\frac{x}{a} = b$ and $\frac{x}{a} + b = c$ using the multiplication principle of equations.	
	2. rewrite an equation involving fractions as an equivalent equation having no fractions by multiplying by the least common multiple of the denominators (use integer denominators, e.g., $3 - \frac{x + 3}{10} = \frac{5x}{4}$).	M9: 230-231 X9-2: 91-93

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

	3. solve and check equations having fractions with numerical denominators.		X9-2: 92-93
	4. solve equations with fractions having monomial (variable) denominators.	M9: 231-233	X9-2: 95-96
	5. multiply by the least common multiple of the denominators to solve equations with binomial denominators..	M9: 232	X9-2: 97
	6. check the solutions of equations involving fractions.		X9-2: 93
H. Verbal Problems Involving Fractions	1. translate the conditions of verbal problems involving fractions into equations.		
	2. solve number problems involving algebraic fractions.	M9: 233	X9-2: 98-100
	3. solve work problems:	M9: 233, 234	X9-2: 101-106
	a. express the part done by each worker per unit of time (the rate of work),		
	b. express the part done by each worker in the time indicated (the product of the rate of work and the number of time units spent working),		
	c. write the equation: the sum of the parts of the job done by each worker is 1.		

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

4. solve motion (rate - time - distance) problems involving fractions.

5. check solutions to all of the above in the conditions of the problems.

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

X. THE REAL NUMBERS

A. Rational Numbers	1. identify the set or sets to which a given number belongs, including: natural (counting) numbers, whole numbers, integers, rational numbers.		
	2. indicate the locations on a number line that correspond to various rational numbers.	M9: 236	X9-2: 115
	3. define the density of the set of rational numbers.	M9: 236-237	X9-2: 115-116
	4. define a rational number as a number that can be written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$.	M9: 235	X9-2: 115
	5. state several names of a rational number (e.g., $2 = 2.0 = \frac{2}{1} = \frac{8}{4}$, etc.)		
	6. express any rational number written as a fraction in its decimal form.		X9-2: 117
	7. state that every fraction and, therefore, every rational number, can be transformed into a terminating decimal or a repeating non-terminating decimal number, and conversely.	M9: 237	X9-2: 117
B. Irrational Numbers	1. define irrational numbers as any number which cannot be expressed as a repeating or terminating decimal, including positives and negatives.	M9: 240, 241	X9-2: 111, 112, 119, 120

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. indicate the approximate location of several irrational numbers on a number line.	M9: 241-242 X9-2: 116
	3. state that every irrational number can be associated with a point on the number line.	M9: 256-257 X9-2: 116
	4. state that the set of numbers including the rational numbers and the irrational numbers is called the set of real numbers.	M9: 240
	5. state that every point on the number line corresponds to a real number.	
C. The Square Root of a Number		
- Fundamental concepts	1. state that one of the two equal factors of a number is called its square root.	M9: 243
	2. use the radical symbol to indicate a square root.	X9-2: 121
	3. write the square roots of perfect squares (e.g., 49, 64, $\frac{4}{9}$, 1.44).	
	4. define the cube root of a number.	X9-2: 120
	5. determine the cube roots of positive and negative numbers which have rational cube roots.	
	6. identify the index of a radical.	X9-2: 121

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	7. find two pairs of equal factors of a number (e.g., 4 = (2)(2) or (-2)(-2).	M9: 244 X9-2: 121-122
	8. state that the principal square root of a number is its positive square root.	X9-2: 122
	9. simplify various expressions including radicals: - $\sqrt{\frac{1}{4}}$, $(\sqrt{16})^2$, $\sqrt{a^2}$, when a = -3, etc.	M9: 244, 245
	10. express $\sqrt{a^2}$ as $\lvert a \rvert$.	
	11. state that the square root of a negative real number is not a real number.	M9: 246
- Approximating square roots	1. distinguish between rational square roots and irrational square roots.	X9-2: 249
	2. round off numbers to the nearest integer, nearest tenth or nearest hundredth.	
	3. locate any positive integer less than 200 between two consecutive perfect squares.	
	4. find square roots of integers less than 200 to the nearest tenth by trial and error.	
	5. find a square root to a specified degree of accuracy using a square root table.	M9: 250-251 X9-2: 252

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

D. The Pythagorean Formula	1. define a right triangle and its parts: hypotenuse, legs, right angle, acute angles.		
	2. solve equations of the form $x^2 = a$ and $x^2 + a = b$.		
	3. state the Pythagorean formula.		X9-2: 108
	4. use the Pythagorean formula to find the length of the hypotenuse or a leg of a right triangle.	M9: 255, 256	X9-2: 110-111
	5. determine whether the lengths of the sides of a given triangle satisfy the Pythagorean formula.	M9: 256	X9-2: 108-109
E. Radicals			
- Simplifying radicals	1. call the expression under the radical sign the radicand.		X9-2: 121
	2. use the principle $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a > 0$ and $b > 0$ to simplify a radical.	M9: 259-260	X9-2: 123-125
	3. state that roots are distributed over multiplication but not over addition ($\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$).	M9: 258	
	4. find the square root of a perfect square monomial (e.g., $\sqrt{x^4}$, $\sqrt{x^6}$, $\sqrt{9x^2y^{10}}$, etc.)	M9: 260	
	5. simplify the square roots of monomials which are not perfect squares (e.g., $\sqrt{x^5}$).	M9: 261, 262	X9-2: 125

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

-Multiplying radicals

1. use the principles that $\sqrt{a}\ \sqrt{b} = \sqrt{ab}$ where $a > 0$ and $b > 0$ and $a\sqrt{b}\ \ c\sqrt{d} = ac\sqrt{bd}$ where $b > 0$ and $d > 0$ to find the product of two irrational numbers. X9-2: 126-127
2. find the product of an irrational number and the sum or difference of two real numbers (e.g., $\sqrt{3}\ (1 - 2\sqrt{5})$).
3. find the second power of an irrational number (e.g., $(2\sqrt{3})^2$).

-Dividing radicals

use the principle $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$, where $a > 0$ and $b > 0$, to find the quotient of two radicals (simplify the quotient where possible). X9-2: 128-129

- Rationalizing a denominator

1. find the product of two equal radicals (e.g., $\sqrt{3}\ \sqrt{3}$).
2. use the multiplicative identity and the principle $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ to rationalize irrational denominators (e.g., $\frac{1}{\sqrt{4}}, \frac{\sqrt{5}}{\sqrt{8}}, \sqrt{\frac{7}{10}}, \frac{3\sqrt{6}}{\sqrt{12}}$). X9-2: 130-132

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
- Combining radicals	1. identify "like" radicals.	X9-2: 133
	2. add radicals using the distributive property: $a\sqrt{b} + c\sqrt{b} = \sqrt{b}\ (a + c)$.	X9-2: 133
	3. combine radicals by first simplifying them to form like radicals.	X9-2: 134-135
- Solving radical equations	1. show that equivalent equations result when both members of a numerical equation are squared (e.g., $5 + 1 = 6$ so $25 + 2(5)(1) + 1 = 6^2$).	
	2. solve simple radical equations by squaring both sides, after isolating the square root (e.g., $\sqrt{x + a} = b$ or $a\sqrt{x + b} = c$).	
	3. state that squaring an equation with a variable may produce equations which are not equivalent.	
	4. check solutions to radical equations.	
- Solving verbal problems where the solution involves irrational numbers	1. translate the conditions of verbal problems into equations, where the solution set contains irrational numbers.	
	2. solve and check verbal problems.	X9-2: 139-141

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

XI. RATIO, PROPORTION AND TRIGONOMETRY				
A. Fundamental Concepts	1.	express the ratio between two quantities a and b as "a to b", a : b, a ÷ b, $\frac{a}{b}$, a percent, and a decimal.	M9: 271-272	X9-1: 96, 98 X9-2: 142-143
	2.	solve a problem involving ratios algebraically by representing quantities in the ratio a:b as ax and bx.	M9: 272, 273, 274	X9-1: 96-97
	3.	define proportion.	M9: 275	X9-2: 145
	4.	find an unknown fourth proportional algebraically, using the rule: if $\frac{a}{b} = \frac{c}{d}$, then ad = bc.	M9: 276-277	X9-2: 145, 146
	5.	solve simple verbal problems involving proportions.		X9-2: 145
B. Similar Triangles	1.	state the definition of similar polygons.	M9: 280-281	X9-2: 146, 147
	2.	state the generalization that two triangles are similar if two angles of one triangle are equal in measure to two angles of the other triangle.	M9: 282, 283	X9-2: 147
	3.	state that if two triangles are similar, then their corresponding sides are in proportion.	M9: 285	X9-2: 146, 148
	4.	recognize visually when two given triangles are similar.	M9: 281, 285, 286	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	5. draw diagrams expressing conditions in verbal problems that involve similar triangles.	X9-2: 146-147, 148-149
	6. use proportions in similar triangles to determine the measures of unknown sides (the method of indirect measurement).	M9: 284, 285 X9-2: 147-149
C. Numerical Trigonometry		
- Fundamental concepts	1. state that if an acute angle of one right triangle is equal in measure to an acute angle of a second right triangle, the right triangles are similar and their sides are in proportion.	M9: 288 X9-2: 152-153
	2. identify the leg adjacent to an acute angle and the leg opposite an acute angle in a right triangle.	X9-2: 151
	3. state that for a given acute angle in several different right triangles the ratios of the leg opposite the angle to the leg adjacent to the angle are equal in value.	M9: 289 X9-2: 152
- Tangent ratio	1. name the ratio of the opposite leg to the adjacent leg the tangent ratio.	
	2. express the tangent ratios for a given right triangle in terms of the sides of the triangle.	M9: 289, 290 X9-2: 153-154
	3. read the table of values of the trigonometric ratios to find the tangent of a given angle.	X9-2: 156

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

4. find an angle correct to the nearest degree, given the value of its tangent. — X9-2: 157, 162

5. solve problems using the tangent ratio, $\tan A = \frac{x}{b}$, $\tan A = \frac{a}{x}$, and $\tan X = \frac{a}{b}$. — M9: 292, 293 X9-2: 156, 157, 158

6. define angle of elevation. — X9-2: 159-160

7. draw a diagram according to the conditions of a problem including an angle of elevation.

8. solve a problem including an angle of elevation by using the tangent ratio.

9. define angle of depression. — X9-2: 160-161

10. determine an angle in a given right triangle by finding the complement of an angle of depression.

11. state that the angle of depression is equal to the angle of elevation. — X9-2: 161

12. draw a diagram according to the conditions of a problem involving an angle of depression.

13. solve a problem including an angle of depression by using the tangent ratio.

14. use the complement of the given acute angle to avoid division when the adjacent leg is to be found. — X9-2: 157

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
- Sine ratio	1. name the ratio of the opposite leg to the hypotenuse the sine ratio.	
	2. express the sine ratios for a given right triangle in terms of the sides of the triangle.	X9-2: 164
	3. find the sine of an angle using the table of values.	M9: 294
	4. find an angle given its sine.	M9: 295 X9-2: 165
	5. draw a diagram according to the conditions of a problem involving the sine ratio.	X9-2: 164-167
	6. solve problems using the sine ratio.	M9: 295 X9-2: 165-167
- Cosine ratio	1. name the ratio of the adjacent leg to the hypotenuse the cosine ratio.	
	2. express the cosine ratios for a given right triangle in terms of the sides of the triangle.	M9: 296 X9-2: 169, 170
	3. find the cosine of an angle using the table of values.	M9: 296
	4. find an angle given its cosine.	M9: 296 X9-2: 171
	5. draw a diagram according to the conditions of a problem involving the cosine ratio.	X9-2: 170-171
	6. solve problems using the cosine ratio.	M9: 297, 298 X9-2: 170-171

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

- Using all three trigonometric ratios	1. write and explain SOHCAHTOA as a device for remembering the definitions of the three trigonometric ratios.		
	2. name the trigonometric ratio needed to solve any right triangle trigonometry problem.	M9: 297-298	X9-2: 173-175
	3. state that the values of the cosine and sine ratios are between 0 and 1, since the hypotenuse is the denominator.		X9-2: 164, 169
	4. state which ratios have increasing values and which have decreasing values as the angle increases.	M9: 291	X9-2: 155, 164, 169

Geometry

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WORK STUDY SKILLS: GEOMETRY

FOLLOWING DIRECTIONS

Taking notes from oral directions requiring several steps.

Applying a pattern for following written directions:

- Reading for overview
- Rereading for specific steps
- Carrying out steps
- Checking for correct completion

Applying skills in following directions to the organization of examination papers (e.g., the display of work, the location of final answers, the calculation of answers to correct degrees of accuracy, etc.)

Discriminating between the instructions "sketch," "draw," and "construct."

Discriminating between the instructions "find," "show," and "prove."

LOCATING INFORMATION

Interpreting and rephrasing questions.

Scanning for specific information; skimming for overview.

Translating key words in given information into more useful forms (e.g., given that two lines are perpendicular, to express this relationship in terms of right angles).

Recognizing the conditions of congruence and parallelism expressed symbolically on diagrams.

Using the table of values of trigonometric ratios to locate the values for specific angles, or, given the value of a ratio, to locate a specific angle.

Identifying the hypothesis (given) and conclusion (to prove) in a conditional statement.

Reading geometric diagrams for the measures of line segments, angles, and arcs.

Referring to notes for previous definitions, postulates and theorems.

RECALLING INFORMATION

Stating previous definitions, postulates, and theorems.

Writing previous formulas.

Performing previously learned constructions.

Using mnemonic devices for remembering relationships (e.g., SOHCAHTOA, Pythagorean triples, special right triangles, etc.)

INTERPRETING INFORMATION

Choosing items of given information to use in writing a deductive proof.

Interpreting information about angle bisectors, midpoints, medians, altitudes, etc., as statements about congruent angles or line segments.

Determining which generalization (postulate or theorem) is illustrated by a particular deductive argument.

Drawing a diagram based upon the information in a verbal description of a geometric figure.

Writing verbal statements of the relationships between geometric figures represented in diagrams.

Using the words congruent, equal, line, ray, line segment, angle, perpendicular, triangle, parallelogram, parallel, circle, degree, arc, right angle, when reading the symbols for these words in verbal statements.

Reading numerical information from given diagrams on the coordinate plane.

Drawing a coordinate geometry graph from given numerical information about the coordinates of points.

Providing examples from real life of specific geometric figures and relationships.

Deciding on the classification of a geometric figure based on verbal or numerical information about its parts,

or

determining the parallelism or perpendicularity of lines or the collinearity of points given information about slopes.

ORGANIZING INFORMATION

Refining skills in note taking.

Maintaining lists of definitions, postulates, theorems, constructions, vocabulary words.

Displaying work associated with the solution of numerical and algebraic problems.

Formulating a plan for the completion of a deductive proof.

Presenting a deductive proof in the two-column format.

Drawing Venn diagrams and tree diagrams to show the relationships between sets of figures.

Presenting a completed geometric construction.

MANIPULATING PHYSICAL MATERIALS

Measuring line segments with a ruler (English and metric).

Measuring an angle with a protractor.

Using a compass and straightedge in geometric constructions.

Using colored pencils, shadings, dotted lines, etc. to highlight the details of a diagram.

Using graph paper in coordinate geometry applications.

NOTE TO THE TEACHER:

The many theorems that are referred to in this guide go far beyond what may be called minimum essentials. They were included, however, to maintain the logical structure and continuity of geometry as much as possible. To help the teacher to determine which theorems students should be expected to prove formally, and which they may be asked to prove informally, either individually or as part of an activity that involves the whole class, we have placed an asterisk (*) next to those theorems that are identified as required theorems in the New York State curriculum guide to Tenth Year Mathematics (1977).

Similarly, we have placed an asterisk next to those constructions that are identified in the New York State guide as fundamental constructions.

(All page references below are to Mathematics: 10th Year, Curriculum Bulletin,1964-65 Series, No. 8, published by the Board of Education of the City of New York. Available from the Publications Sales Office, Board of Education. 110 Livingston Street, Brooklyn, N.Y. 11201.)

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
I. INTRODUCTION TO GEOMETRY	1. define geometry.	7
A. Origins	2. state briefly the historical origins of geometry in ancient times.	
	3. recall the names of Euclid and Pythagoras as major contributors to the development of geometry.	
B. Basic Definitions and Notoation	1. state the need for undefined terms in the construction of a technical vocabulary.	8
	2. list some undefined terms in geometry: point, line, plane, figure, etc.	8
	3. state criteria for a good definition.	10, 11
	4. identify examples of poor definitions in a variety of fields.	
	5. define line segment, ray, and angle.	11, 12
	6. define the vertex and sides of an angle.	
	7. name lines, rays and line segments using two letters and the appropriate symbol $\leftrightarrow$, $\rightarrow$, or $—$.	10
	8. distinguish between $\overline{AB}$ (line segment AB) and AB (the measure of, or length of line segment AB).	

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CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	9. name an angle using three letters, one letter (the vertex), a numeral or a variable.	11, 12
	10. distinguish between $\measuredangle ABC$ (angle ABC) and $m\measuredangle ABC$ (the measure of angle ABC).	13
	11. measure an angle with a protractor.	
	12. state that the sum of the measures of the angles about a point is 360^{o}.	
	13. define acute, right, obtuse, straight, and reflex angles in terms of their degree measures.	
	14. classify given angles as acute, right, obtuse, straight, or reflex.	
C. Angle Pairs	1. define adjacent, complementary, supplementary, and vertical angles.	13, 14, 15
	2. identify adjacent, complementary, supplementary, and vertical angles in given diagrams.	15
	3. solve algebraic problems involving complementary and supplementary angles.	16
D. Congruence and Related Concepts	1. define congruent line segments and angles as line segments and angles with equal measures.	
	2. use the symbol $\cong$ to indicate congruent line segments and angles.	21

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	3. define the midpoint of a line segment.	
	4. translate a statement about the midpoint of a line segment into a statement about two congruent line segments, and vice versa (e.g., if M is the midpoint of $\overline{AB}$, then $\overline{AM} \cong \overline{MB}$).	31
	5. define bisector of a line segment.	
	6. draw several bisectors of the same line segment.	
	7. translate a statement about the bisector of a line segment into a statement about two congruent line segments, and vice versa.	31
	8. define perpendicular lines.	16
	9. use the symbol $\perp$ to indicate perpendicular lines.	
	10. translate a statement about perpendicular lines into a statement about right angles.	31
	11. define the distance from a point to a line.	
	12. define the perpendicular bisector of a line segment.	
	13. translate a statement about a perpendicular bisector into a statement about right angles and congruent line segments.	
	14. define the bisector of an angle.	

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CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	15. translate a statement about the bisector of an angle into a statement about two congruent angles, and vice versa.	
	16. indicate congruent line segments, congruent angles and right angles on a diagram using the appropriate symbols.	
E. Basic Concepts Regarding Triangles	1. define polygon.	
	2. name a given polygon in terms of the number of its sides (from 3 to 12 sides).	
	3. define triangle.	
	4. identify the six parts of a given triangle.	
	5. define scalene, isosceles, and equilateral triangles according to the number of congruent sides.	
	6. classify given triangles as scalene, isosceles, or equilateral, given information about their sides.	
	7. define acute, right, obtuse and, equiangular triangles in terms of the measures of their angles.	
	8. classify given triangles as acute, right, obtuse, and equiangular, given information about their angles.	
	9. classify triangles according to their sides and angles (e.g., right-isosceles, obtuse-scalene, etc.)	19

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

10. draw sample triangles of given types using straight-edge and protractor.

11. define the median, altitude, and angle bisector of a triangle.

12. draw all three medians, altitudes, or angle bisectors in any triangle (including a right triangle and an obtuse triangle).

13. translate a statement about the median, altitude, or angle bisector of a triangle into a statement about congruent line segments, right angles, or congruent angles.

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
II. THE POSTULATIONAL SYSTEM		
A. Basic Concepts	1. define postulate (axiom).	13
	2. provide examples of postulates (axioms) from algebra and areas outside of mathematics.	
	3. describe deductive reasoning as the process of reaching a conclusion which is the logical outcome of previously accepted generalizations.	1, 2, 3, 4, 9-10
	4. describe inductive reasoning as the acceptance of a generalization after its illustration by examples.	9-10
	5. provide examples of deductive and inductive reasoning in mathematics and in areas outside of mathematics.	9, 10
	6. compare generalizations arrived at deductively and inductively in terms of their certainty.	9
	7. identify the "if" part of an "if...,then" statement as its hypothesis and the "then" part as its conclusion.	15, 17, 18
	8. define theorem as a generalization arrived at deductively.	14
	9. restate a conditional statement in the "if..., then" form if it is not given as such.	14, 17
B. The Postulates of Equality	1. define postulational system.	23, 24, 29
	2. state the postulates of equality: substitution, reflexive, symmetric, transitive, addition, subtraction, multiplication, doubles, division, halves, roots and powers.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	3. state the "wholes" postulate (a quantity is equal to the sum of its parts).	
	4. state the postulate: one and only one line may be drawn between two points.	
	5. state the postulate: the shortest distance between two points is the line segment joining them.	
	6. illustrate the postulates of equality in the solution of simple algebraic equations.	
	7. identify which postulate(s) of equality are illustrated by given deductive arguments about line segments and angles.	28
	8. provide the statements and supporting postulates of equality to complete deductive arguments about line segments and angles.	
C. Proving Simple Angle Theorems	1. describe a formal two-column proof as consisting of a diagram, a statement of the given information, a statement of the desired conclusion, statements and supporting, previously accepted generalizations.	30, 31
	2. prove the following angle theorems:	15, 16, 17
	a. All right angles are congruent. b. All straight angles are congruent. c. If two adjacent angles have their non-common sides on the same straight line, then they are supplementary. d. If two angles are complementary (or supplementary) to the same or congruent angles, then they are congruent. e. If two angles are congruent, then their complements (or supplements) are congruent. f. Vertical angles are congruent.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	3. apply the angle theorems to arrive at conclusions in proof exercises (not proofs of theorems).	
III. CONGRUENT TRIANGLES	1. define congruent triangles.	20
	2. identify six pairs of parts (three pairs of sides and three pairs of angles) that must be congruent if the triangles are to be congruent.	20
	3. identify pairs of congruent sides and angles in congruent triangles as corresponding parts.	22, 23
	4. state that the corresponding sides are opposite the corresponding angles, and vice versa.	
	5. state the SSS postulate.	22
	6. prove triangles congruent by using the SSS postulate.	22
	7. state the SAS postulate.	22
	8. prove triangles congruent by using the SAS postulate.	22
	9. state the ASA postulate.	22
	10. prove triangles congruent by using the ASA postulate.	22
	11. select the appropriate method (SSS, SAS or ASA) to prove triangles congruent in given exercises.	
	12. prove triangles congruent given information about medians, altitudes or angle bisectors.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	13. explain why AAA can not be used to prove triangles congruent.	48
	14. explain why SSA (ASS) can not be used to prove triangles congruent.	48
	15. restate the definition of congruent triangles as: corresponding parts of congruent triangles are congruent.	22, 23
	16. use the relationship between corresponding parts of congruent triangles to prove line segments and angles congruent.	23
	17. draw a diagram according to the conditions of a verbally stated problem involving congruent triangles.	
	18. prove overlapping triangles congruent by any of the three methods.	32
	19. prove triangles congruent in "double congruence" exercises.	32

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
IV. INTRODUCTION TO CONSTRUCTIONS	1. define a construction as a drawing made with a straight-edge and compass.	25
	2. state the following postulates related to constructions: - through a given point any number of lines may be drawn - a line may be extended indefinitely - a given line segment may be laid off any number of times on a given line	
	3. locate the midpoint of a given line segment by construction (bisect a line segment).	
	4. state the postulate: a line segment has one and only one midpoint.	
	5. construct the median in a triangle.	
	6. construct a line segment equal in measure to a given line segment, an integral multiple of the measure of a given line segment, or a multiple involving the fraction ½.	
	7. construct an angle equal in measure to a given angle (copy an angle).	27
	*8. construct the bisector of an angle.	26, 33
	9. state the postulate: an angle has one and only one bisector.	
	10. construct an angle equal in measure to an integral multiple of the measure of a given angle, or a multiple involving the fraction ½.	27, 29

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	11. explain the constructions for copying an angle, bisecting a line segment and bisecting an angle in terms of congruent triangles.	26, 27
	12. construct a triangle congruent to a given triangle by SSS, SAS or ASA.	34
V. PROPERTIES OF THE ISOSCELES TRIANGLE	1. name the parts of an isosceles triangle: base, legs, base angles, vertex angle.	
	*2. prove the theorem: if two sides of a triangle are congruent, then the angles opposite them are congruent (the base angles of an isosceles triangle are congruent).	27-28
	3. use the "base angles theorem" to solve algebraic problems about isosceles triangles.	
	4. use the "base angles theorem" to provide pairs of congruent angles as part of congruence proofs, including problems which are expressed verbally and no diagram is provided.	
	5. prove that an equilateral triangle is equiangular.	44
	6. prove that the bisector of the vertex angle of an isosceles triangle bisects the base and is perpendicular to the base.	33

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
VI. PROVING LINES PERPENDICULAR	1. prove that if two adjacent angles are congruent and supplementary, then they are right angles.	33
	2. use the above theorem to prove lines perpendicular in proof exercises.	
	* 3. construct a line perpendicular to a given line at a given point on it (erecting a perpendicular).	33
	4. state the postulate: one and only one line may be drawn perpendicular to a given line at a given point on the line.	33
	5. explain the construction for erecting a perpendicular in terms of congruent triangles and adjacent right angles.	
	* 6. construct a line perpendicular to a given line from a given point not on the given line (dropping a perpendicular).	33
	7. construct an altitude in a triangle, including an obtuse triangle.	
	8. explain the above constructions in terms of congruent triangles and adjacent right angles.	
	9. define the distance from a point to a line.	46
	10. state the postulate: the shortest distance from a point to a line is the length of the perpendicular line segment from the point to the line.	
	11. construct the perpendicular bisector of a line segment.	
	12. prove that the median to the base of an isosceles triangle is perpendicular to the base.	

CONTENT AREA AND TOPIC		PERFORMANCE OBJECTIVES	REFERENCES
		The student will be able to ...	
VII. PARALLEL LINES			
A. Basic Concepts	1.	define parallel lines.	37
	2.	indicate parallel lines on a diagram using the "arrow" symbols.	
	3.	provide examples of parallel lines from real life.	37
	4.	define a transversal.	38
	5.	define pairs of corresponding, alternate interior and consecutive interior angles.	38
	6.	name pairs of corresponding, alternate interior and consecutive interior angles in a diagram containing two lines (parallel or not) and one transversal.	38
B. Properties of Parallel Lines	1.	state the postulate: if parallel lines are cut by a transversal, then the alternate interior angles formed are congruent.	41
	2.	use the above postulate and simple angle theorems to prove the theorems:	41, 42
		a. If parallel lines are cut by a transversal, then the corresponding angles formed are congruent.	
		b. If parallel lines are cut by a transversal, then consecutive interior angles formed are supplementary.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	3. use the properties of parallel lines to solve numerical and algebraic problems involving the measures of angles.	40 42
	4. use the properties of parallel lines to provide pairs of congruent or supplementary angles in congruence proof exercises.	
	5. draw a diagram illustrating the conditions of a verbally stated problem involving parallel lines.	
C. Proving Lines Parallel	1. form the converse of a conditional statement (if...then) by interchanging its hypothesis and conclusion.	41
	2. provide examples, from within mathematics and outside of mathematics, in which true statements have false converses, true statements have true converses, false statements have true converses and false statements have false converses.	42, 43
	3. conclude that the truth or falsity of the converse of a statement is independent of the truth or falsity of the statement itself.	42
	4. write the converses of the three generalizations concerning the properties of parallel lines.	37, 38
	5. identify the statement "if the alternate interior angles formed by two lines and a transversal are congruent, then the lines are parallel" as a postulate.	38
	6. use the above postulate to prove the remaining two converses.	38, 39

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	7. accept either one of the latter two generalizations as a postulate and use it to prove the remaining two.	38
	8. use the three generalizations to prove that lines are parallel in proof exercises.	39
	9. state Euclid's parallel postulate: through a point not on a given line, one and only one line can be drawn parallel to the given line.	37
	* 10. construct a line parallel to a given line through a given point not on it.	38
	11. prove that if two lines are parallel to the same line, then they are parallel to each other.	38
	12. prove that if two lines are perpendicular to the same line, then they are parallel to each other.	39

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
VIII. THE SUM OF THE MEASURES OF THE ANGLES OF A TRIANGLE	1. measure the angles of triangles and find the sums of the measures.	43
	2. state a tentative generalization concerning the sum of the measures of the angles of a triangle.	43
	* 3. prove the "angle sum theorem" using concepts involving parallel lines.	44
	4. apply the "angle sum theorem" to solve numerical and algebraic problems about the angles of a triangle.	
	5. prove the following corollaries of the "angle sum theorem":	44
	a. If two triangles agree in two angles, then they agree in their third angles.	
	b. The acute angles of a right triangle are complementary.	
	c. The measure of each angle of an equilateral triangle is 60^{o}.	
	d. The measure of each acute angle of an isosceles-right triangle is 45^{o}.	
	e. An exterior angle of a triangle is equal in measure to the sum of the measures of the two remote interior angles.	
	f. The sum of the measures of the angles of a quadrilateral is 360^{o}.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	6. use the corollaries of the "angle sum theorem" to solve a variety of numerical and algebraic problems about the angles of triangles and quadrilaterals.	44
	7. divide a polygon of any number of sides into non-overlapping triangles.	123
	8. state a rule for determining the number of triangles possible in any polygon, given the number of sides.	123, 124
	9. state a rule for determining the sum of the measures of the interior angles of any polygon, given the number of sides.	123

CONTENT AREA AND TOPIC		PERFORMANCE OBJECTIVES	REFERENCES
		The student will be able to ...	
IX. PROVING TRIANGLES CONGRUENT BY SAA AND HY-LEG			
A. SAA	1.	prove the theorem: two triangles are congruent if they agree in two angles and a non-included side (SAA).	46-47
	2.	use the method of SAA to prove triangles congruent in proof exercises.	46-47
	* 3.	use the method of SAA to prove the converse of the "base angles theorem."	47
	4.	list the logical sequence of theorems and postulates that led to the proof of the converse of the "base angles theorem".	47
	5.	use the converse of the "base angles theorem" to prove line segments congruent and triangles isosceles in proof exercises.	
B. Hy-Leg	* 1.	prove the theorem: two right triangles are congruent if they agree in their hypotenuses and one leg.	48-49
	2.	differentiate between the method of hy-leg and the other four methods of proving triangles congruent in that hy-leg is applicable only to a sub-set of the set of triangles.	
	3.	use hy-leg to prove triangles congruent in proof exercises.	49

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
X. QUADRILATERALS		
A. Properties of the Parallelogram	1. define parallelogram.	52
	2. use the symbol $\square P$ or $\square$ to designate a parallelogram.	
	3. prove the following theorems about the properties of parallelograms:	55, 56
	a. A diagonal of a parallelogram divides it into two congruent triangles.	
	b. The opposite sides of a parallelogram are congruent.	56
	c. The opposite angles of a parallelogram are congruent.	
	d. Any pair of consecutive angles of a parallelogram are supplementary.	
	e. The diagonals of a parallelogram bisect each other.	
	4. make a list of six properties of parallelograms (including opposite sides parallel).	53, 55
	5. use the properties of the parallelogram to solve a variety of numerical and algebraic problems about the sides, angles and diagonals of parallelograms.	56
	6. use the properties of parallelograms to answer a variety of true-false, multiple choice and completion questions about quadrilaterals.	

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CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	7. use the properties of parallelograms as reasons in proof exercises.	
B. Proving That a Quadrilateral is a Parallelogram	1. use the definition of a parallelogram to prove that a given quadrilateral is a parallelogram.	57
	2. prove the following theorems about parallelograms:	57, 58
	a. If both pairs of opposite sides of a quadrilateral are congruent, then it is a parallelogram.	
	b. If both pairs of opposite angles of a quadrilateral are congruent, then it is a parallelogram.	
	c. If one pair of sides of a quadrilateral are congruent and parallel, then it is a parallelogram.	
	d. If the diagonals of a quadrilateral bisect each other, then it is a parallelogram.	
	3. provide examples of quadrilaterals (the "kite") that are not parallelograms and yet are divided into two congruent triangles by a diagonal.	
	4. use the above theorems to prove that given quadrilaterals are parallelograms.	57
C. Properties of the Rectangle	1. define rectangle.	52
	2. describe the relationships between the set of quadrilaterals, the set of parallelograms and the set of rectangles by drawing a Venn diagram or tree diagram.	52, 54, 55

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	3. prove the following theorems about rectangles: a. A rectangle is equiangular. b. Each angle of a rectangle is a right angle. c. The diagonals of a rectangle are congruent.	
	4. list all the properties of a rectangle, including all the properties of parallelograms in general.	55
	5. use the properties of a rectangle to solve a variety of numerical and algebraic problems.	
	6. use the properties of the rectangle to answer a variety of true-false, multiple choice and completion questions.	
	7. use the properties of rectangles in proof exercises.	59
D. Proving That a Quadrilateral Is a Rectangle	1. prove that a given quadrilateral is a rectangle by first proving that it is a parallelogram and then proving that it has a right angle (the definition of rectangle).	
	2. prove the theorem: a quadrilateral is a rectangle if it is equiangular.	
	3. prove the theorem: a quadrilateral is a rectangle if it is a parallelogram with congruent diagonals.	
E. Properties of the Rhombus	1. define rhombus.	53

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. describe the relationships between the set of quadrilaterals, the set of parallelograms, the set of rectangles and the set of rhombuses by drawing a Venn diagram or a tree diagram.	52, 54, 55
	3. prove the following theorems about rhombuses:	55
	a. A rhombus is equilateral.	
	b. The diagonals of a rhombus are perpendicular.	
	c. The diagonals of a rhombus bisect its angles.	
	d. The diagonals of a rhombus divide it into four congruent triangles.	
	4. list all the properties of a rhombus, including all the properties of parallelograms in general.	55
	5. use the properties of a rhombus to solve a variety of numerical and algebraic problems.	56
	6. use the properties of a rhombus to answer a variety of true-false, multiple choice and completion questions.	
	7. use the properties of a rhombus in proof exercises.	
F. Proving That a Quadrilateral Is a Rhombus	1. prove that a given quadrilateral is a rhombus by first proving that it is a parallelogram and then proving that two adjacent sides are congruent (the definition of rhombus).	55
	2. prove the theorem: an equilateral parallelogram is a rhombus.	55

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	3. prove the theorem: a parallelogram with perpendicular diagonals is a rhombus.	55
	4. provide examples of quadrilaterals (the "kite" that are not rhombuses and yet have perpendicular diagonals.	
	5. use the above theorems to prove that given quadrilaterals are rhombuses in proof exercises.	
	6. define rhomboid as a non-equilateral and non-equiangular parallelogram.	
G. Properties of the Square	1. define square as a special kind of rectangle.	53
	2. define square as a special kind of rhombus.	53
	3. describe the relationships between the set of quadrilaterals, the set of parallelograms, the set of rhomboids, the set of rectangles, the set of rhombuses and the set of squares by drawing a Venn diagram or a tree diagram.	52, 54, 55
	4. list all the properties of the square, including all the properties of the parallelogram in general, all the properties of the rectangle and all the properties of the rhombus.	55
	5. identify the square as the type of quadrilateral with the greatest number of special properties.	
	6. use the properties of squares in a variety of numerical and algebraic problems.	

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CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	7. use the properties of squares to answer a variety of true-false, multiple choice and completion questions.	
	8. use the properties of squares in proof exercises.	
H. Proving That a Quadrilateral Is a Square	1. prove that a quadrilateral is a square by first proving that it is a rectangle and then proving that it has two congruent adjacent sides (a definition of square).	
	2. prove that a quadrilateral is a square by first proving that it is a rhombus and then proving that it has a right angle (a definition of square).	
I. Properties of the Trapezoid	1. define trapezoid.	52
	2. add the set of trapezoids to previously drawn Venn diagrams or tree diagrams to show its relationship to the other sets of quadrilaterals.	
	3. list the properties of the trapezoid.	
	4. use the properties of the trapezoid to solve a variety of numerical and algebraic problems.	
	5. use the properties of the trapezoid to answer a variety of true-false, multiple choice and completion questions.	
	6. use the properties of the trapezoid in proof exercises.	
	7. define isosceles trapezoid.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	8. add the set of isosceles trapzoids to previously drawn Venn diagrams or tree diagrams to show its relationship to the other sets of quadrilaterals.	52, 54 55
	9. prove the following theorems about isosceles trapezoids:	60
	a. The base angles of an isosceles trapezoid are congruent.	
	b. The diagonals of an isosceles trapzoid are congruent.	
	10. use the properties of the isosceles trapezoid to solve numerical and algebraic problems.	60
	11. use the properties of the isosceles trapezoid to answer a variety of true-false, multiple choice, and completion questions.	
	12. use the properties of the isosceles trapezoid in proof exercises.	
	13. define trapezium as a quadrilateral with no pairs of parallel sides.	
	14. add the set of trapeziums to complete the Venn diagram or tree diagram of quadrilaterals.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XI. THREE OR MORE PARALLEL LINES	1. prove that if three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on any transversal.	84
	* 2. divide a given line segment into any number of congruent segments by construction.	84
	3. define the median of a trapezoid.	
	4. prove the theorem: the median of a trapezoid is parallel to the bases and equal in length to the average of their lengths.	
	5. prove that if a line bisects one side of a triangle and is parallel to a second side, then it bisects the third side.	90
	6. use the above theorems to solve numerical and algebraic problems about trapezoids and triangles.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XII. INTRODUCTION TO CIRCLES		
A. Basic Concepts	1. define circle.	67
	2. define radius, center, chord, diameter, arc, major arc, minor arc, semicircle, intercepted arc, central angle.	
	3. define equal circles.	
	4. state the relationship between the length of a radius and the length of a diameter.	
	5. choose a hoop as a better model of a circle than a disc.	
	6. state that there are 360 arc-degrees in a circle.	
	7. use the unit arc-degree to describe the size of arcs which are expressed as fractions of an entire circle.	68
	8. describe the length of an arc in linear units.	
	9. distinguish between the degree measure and the length of an arc.	68
	10. draw examples of arcs which are equal in degree measure but unequal in length, or equal in length and unequal in degree measure.	67

CONTENT AREA AND TOPIC		PERFORMANCE OBJECTIVES	REFERENCES
		The student will be able to ...	
B. Central Angles, Arcs and Chords	1.	distinguish between angle-degrees and arc-degrees as different units (as weight pounds and British money pounds are different units).	68
	2.	compare the apparent degree measures of arcs intercepted by central angles of 180^{o}, 90^{o} and 45^{o} to the measures of the central angles.	
	3.	prove that in the same or equal circles, congruent chords have congruent central angles.	
	4.	state the postulate: a central angle is equal in degrees to its intercepted arc.	68, 76
	5.	prove the theorem: in the same or in equal circles, congruent central angles intercept congruent arcs.	68
	6.	prove the theorem: in the same or in equal circles, congruent arcs have congruent chords (and the converse).	69
	7.	solve numerical and algebraic problems about the arcs of a circle, their chords and central angles.	70
	8.	use the relationships between arcs, chords and central angles in proof exercises.	68
C. A Diameter Perpendicular to a Chord	* 1.	use the relationship between central angles and arcs to prove the theorem: a diameter perpendicular to a chord bisects the chord and its major and minor arcs.	69
	2.	prove the converses of the above theorem.	70
	3.	use the above theorems in proof exercises.	
	* 4.	locate by construction the center of a given circle or arc.	70

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	5. define inscribed polygons and circumscribed circles.	
	* 6. locate by construction the center of the circle circumscribed about a given triangle.	
	7. prove the theorem: in the same circle, congruent chords are equidistant from the center.	72
	8. prove the theorem: in the same circle, chords that are equidistant from the center are congruent.	72
	9. use the above theorems in proof exercises.	
	10. list a logical sequence of theorems used to prove the above "equidistant" thoerems.	69, 71
D. Tangents to a Circle; Indirect Proof	1. define tangent to a circle.	73
	2. define secant.	
	3. define indirect proof (methods of contradiction and elimination).	140-141
	4. provide examples of indirect proof in argumentative conversation ("O.K., let's say you're right for a minute ...").	
	5. use indirect proof to prove the theorem: a line tangent to a circle is perpendicular to the radius at the point of contact.	73
	6. use the above theorem in proof exercises.	
	7. prove the converses of the above theorem.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	* 8. construct a line tangent to a given circle at a given point on the circle.	73
	9. prove the theorem: if two tangents are drawn to the same circle from the same external point, then they are congruent.	74
	10. use the above theorem in numerical problems and in proof exercises.	74
	11. define common external tangents, common internal tangents, tangent circles and line of centers.	
	12. state the number of common tangents two circles have when they are separate from each other (neither is contained inside the other), externally tangent to each other, intersecting in two points, internally tangent to each other, or not touching (with one contained inside the other).	
	13. prove the theorem: if two circles intersect in two points, then their line of centers is the perpendicular bisector of their common chord.	
E. Angle Measurement in a Circle		
- Inscribed angles	1. define inscribed angle.	
	* 2. prove that an inscribed angle is measured by ½ its intercepted arc (three cases).	76, 77
	3. solve numerical and algebraic problems about inscribed angles, using the above relationship.	77

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	4. prove the following corollaries: a. In the same or in equal circles, congruent inscribed angles intercept congruent arcs. b. In the same or in equal circles, inscribed angles that intercept the same or congruent arcs are congruent. c. In a circle, parallel chords cut off congruent arcs between them. d. An angle inscribed in a semicircle is a right angle. e. If a quadrilateral is inscribed in a circle, its opposite angles are supplementary.	
	5. use the above corollaries to solve algebraic and numerical problems.	77
	6. use the above corollaries in proof exercises.	
	* 7. construct a line tangent to a given circle from a given external point.	79
- Tangent-chord angles	1. prove the theorem: an angle formed by a tangent and a chord is measured by ½ its intercepted arc.	76
	2. use the above theorem in numerical and algebraic problems.	

CONTENT AREA AND TOPIC		PERFORMANCE OBJECTIVES	REFERENCES
		The student will be able to ...	
- Angles formed by two chords intersecting inside a circle	* 1.	prove that an angle formed by two chords intersecting inside a circle is measured by ½ the sum of its intercepted arcs.	76
	2.	describe the method of calculating the measure of a chord-chord angle as averaging the measures of the two arcs.	76
	3.	apply the above theorem to solve numerical and algebraic problems.	
	4.	show that the special case in which the two chords are diameters (a central angle) the average of the two measures is equal to one of the measures.	78
- Angles formed by tangents and secants	* 1.	prove the theorems: an angle formed by two tangents, two secants or a tangent and a secant is measured by ½ the difference of intercepted arcs.	76
	2.	use the above theorems to solve algebraic and numerical problems.	
- Summary	1.	list seven types of angles formed in circles and the ways in which they are measured.	76
	2.	solve complex numerical and algebraic problems in angle measurement involving several types of angles.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XIII. SIMILARITY		
A. Proportional Line Segments	1. define a ratio as a quotient comparing two numbers.	81
	2. define a proportion as an equation stating that two ratios are equal.	81
	3. identify the four proportionals, the means, the extremes and the mean proportional in a proportion.	81
	4. write ratios and proportions to express relationships between the measures of angles and line segments in verbal problems.	
	5. rewrite a given proportion as an equivalent proportion using the methods of inversion, alternation, addition and subtraction.	83
	6. prove the theorem: in a proportion, the product of the means is equal to the product of the extremes.	81
	7. solve a proportion for an unknown fourth or mean proportional.	82
	8. prove the theorem: if a line divides two sides of a triangle proportionally, then it is parallel to the third side.	89
	9. use the above theorem to determine whether lines are parallel, given the measures of line segments in triangles.	
	10. prove the theorem: three or more parallel lines intercept proportional segments on any two transversals.	84

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	11. prove the theorem: if a line is parallel to one side of a triangle and intersects the other two, then it divides the other two sides proportionally.	82, 84
	12. use the above theorem to solve numerical and algebraic problems.	
	13. define similar polygons.	80
	14. define ratio of similitude.	
	15. provide examples of similarity from real life.	
B. Proving Triangles Similar	* 1. prove the theorem: two triangles are similar if they agree in three angles (AAA).	85
	2. prove the theorem: two triangles are similar if they agree in two angles (AA).	85
	3. use the method of AA to prove triangles similar in proof exercises.	85
	4. name pairs of corresponding sides in given similar triangles.	86, 87
	* 5. construct a triangle similar to a given triangle, using a given line segment as the base.	
	6. write proportions relating the measures of sides of similar triangles (including extended proportions).	81, 83

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	7. solve numerical and algebraic problems about the corresponding sides of similar triangles.	83
	8. prove that measures of line segments are in proportion by proving triangles similar and then using the definition of similarity (corresponding sides of similar triangles are in proportion).	86
	9. prove that products of measures of line segments are equal by first proving that triangles are similar, then that measures of line segments are proportional, and then using the theorem that in a proportion, the product of the means is equal to the product of the extremes.	87
	10. prove the theorem: two triangles are similar if an angle of one is congruent to an angle of the other and the sides including these angles are in proportion.	89
	11. prove the theorem: two triangles are similar if three sides of one are proportional to three sides of the other.	93-94
	12. prove the theorem: if a line is drawn parallel to one side of a triangle and intersects the other two sides, it cuts off a triangle similar to the given triangle.	86
	13. use the above theorems to prove triangles similar in proof exercises.	
C. Proportions Involving Chords, Tangents and Secants	1. prove the theorem: if two chords intersect within a circle, the product of the measures of the segments of one chord is equal to the product of the measures of the segments of the other chord.	88

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. draw examples in which the products of the measures of the segments are equal but the measures of the chords themselves are not equal.	88
	3. use the theorem to solve numerical and algebraic problems about intersecting chords.	88
	4. prove the theorem: if a tangent and a secant are drawn to the same circle from the same external point, then the length of the tangent is the mean proportional between the length of the secant and the length of its external segment.	
	5. write proportions expressing the relationship between the measures of line segments in verbal problems about tangents and secants.	88
	6. solve numerical and algebraic problems about tangents and secants drawn to the same circle.	88
	7. prove the theorem: if two secants are drawn to the same circle from the same external point, then the product of the length of one secant and the length of its external segment is equal to the product of the length of the other secant and the length of its external segment.	88
	8. write equations expressing the relationship between the measures of line segments in verbal problems about secants.	
	9. solve numerical and algebraic problems about secants drawn to a circle.	

CONTENT AREA AND TOPIC		PERFORMANCE OBJECTIVES	REFERENCES
		The student will be able to ...	
D. Mean Proportionals in Right Triangles	1.	define the projection of a line segment.	96
	2.	name the projections of line segments in given diagrams.	
	* 3.	prove the theorems: if the altitude is drawn to the hypotenuse of a right triangle, then a. the two triangles formed are similar to the given triangle and to each other, b. the length of the altitude is the mean proportional between the lengths of the segments (projections of the legs) of (on) the hypotenuse, and c. the length of either leg is the mean proportional between the length of the hypotenuse and the length of the projection of that leg on the hypotenuse.	96
	4.	solve numerical and algebraic problems about an altitude drawn to the hypotenuse of a right triangle.	
E. The Pythagorean Theorem	1.	recall the Pythagorean formula from work done in algebra.	99-100
	2.	state the relationship expressed by the Pythagorean formula verbally.	
	* 3.	prove the Pythagorean theorem.	100
	4.	recall and apply the method to square irrational numbers.	
	5.	use the Pythagorean theorem to solve a variety of numerical and algebraic problems, including rational and irrational answers.	100

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CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	6. state combinations of integers that satisfy the Pythagorean formula: 3, 4, 5 ; 5, 12, 13 ; 8, 15, 17 ; 7, 24, 25 and their multiples (Pythagorean triples).	
	7. recognize when, in numerical problems, the sides of a right triangle are Pythagorean triples, and use this fact to solve the problems.	
	8. prove, by indirect proof, the converse of the Pythagorean theorem.	104
	9. determine whether or not a given triangle is a right triangle by determining whether the measures of its sides satisfy the Pythagorean formula.	105
F. Special Right Triangles	1. draw an equilateral triangle and show how an altitude divides it into two 30^o - 60^o - 90^o triangles.	102-103
	2. compute the measures of the sides of a 30^o - 60^o - 90^o triangle, given the measures of the sides of the equilateral triangle.	
	3. generalize the following relationships about the measures of the sides of a 30^o - 60^o - 90^o triangle:	103
	a. The measure of the side opposite the 30^o angle is $\frac{1}{2}$ the measure of the hypotenuse.	
	b. The measure of the side opposite the 60^o angle is $\frac{1}{2}$ the measure of the hypotenuse multiplied by $\sqrt{3}$.	
	c. the length of the longer leg is the length of the shorter leg multiplied by $\sqrt{3}$.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	4. use the above relationships to solve a variety of numerical and algebraic problems, including rational and irrational answers.	103
	5. draw a square and show how a diagonal divides it into two 45^o -45^o - 90^o triangles (isosceles-right triangles).	103
	6. compute the lengths of the sides of a 45^o - 45^o - 90^o triangle, given the length of the sides of the square.	103
	7. generalize the following relationships about the sides of a 45^o - 45^o - 90^o triangle:	
	a. the length of each leg is $\frac{1}{2}$ the length of the hypotenuse multiplied by $\sqrt{2}$.	
	b. the length of the hypotenuse is the length of the leg multiplied by $\sqrt{2}$.	
	8. use the above relationships to solve a variety of numerical and algebraic problems, including rational and irrational answers.	
G. Trigonometry of the Right Triangle	1. state that if two right triangles agree in one acute angle, then they are similar and their corresponding sides are in proportion.	107
	2. identify the leg adjacent to and the leg opposite from a given acute angle in a right triangle.	106
	3. state that for a given acute angle in several different right triangles, the ratios of the leg opposite to the	106

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	leg adjacent to the angle are equal in value.	
	4. name the ratio of the opposite leg to the adjacent leg the tangent ratio.	107
	5. express the tangent ratios in given right triangles in terms of the lengths of the sides of the triangles.	
	6. read the table of values of the trigonometric ratios to find the tangent of a given angle.	107
	7. find the measure of an angle correct to the nearest degree, given the value of its tangent either as a four-place decimal or as a fraction.	107
	8. define angles of elevation and depression.	108
	9. use the measure of an angle of depression to determine the measures of angles in the triangle.	108
	10. draw diagrams to represent the conditions of verbal problems, including angles of elevation and depression.	
	11. solve a variety of numerical problems using the tangent ratio, including finding the opposite leg, adjacent leg and the measure of the angle.	107, 109
	12. reinterpret a problem to avoid using division when tangent is to be used to find the adjacent leg.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	13. define the sine ratio.	107
	14. express the sine ratios in given right triangles in terms of the lengths of the sides of the triangles.	
	15. read the table of values of the trigonometric ratios to find the sine of a given angle.	108
	16. find the measure of an angle correct to the nearest degree, given the value of its sine either as a four-place decimal or a fraction.	
	17. draw diagrams to represent the conditions of problems which involve the sine ratio.	
	18. solve a variety of numerical problems using the sine ratio.	
	19. define the cosine ratio.	107
	20. express the cosine ratios in given right triangles in terms of the lengths of the sides of the triangles.	
	21. read the table of values of the trigonometric ratios to find the cosine of a given angle.	
	22. find the measure of an angle correct to the nearest degree, given the value of its cosine either as a four-place decimal or a fraction.	108
	23. draw diagrams to represent the conditions of problems which involve the cosine ratio.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

24. write equations to solve numerical problems about the sides of right triangles, both in terms of the sine of one acute angle and the cosine of its complement.

25. write and explain SOHCAHTOA as a device for remembering the definitions of the three trigonometric ratios.

26. name the appropriate trigonometric ratio needed to solve any right triangle problem. 109

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XIV. COORDINATE GEOMETRY		
A. Basic Concepts	1. recall the meaning of the terms: coordinate axes, origin, quadrants, coordinates, abscissa and ordinate.	62
	2. draw coordinate axes on graph paper and plot points, given their coordinates.	62
	3. determine the coordinates of a point, given its location on the coordinate plane.	
B. Midpoints	1. determine the coordinates of the midpoint of a line segment, given the coordinates of its endpoints.	63
	2. generalize that the coordinates of the midpoint of the line segment whose endpoints are (a, b) and (c, d) are $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$.	63, 90
	3. show that a line segment is parallel to one side of a triangle by showing that its endpoints are the midpoints of the other two sides.	
	4. use the midpoint formulas in a variety of problems to calculate the coordinates of midpoints or endpoints.	64
D. Distances	1. determine the distance between two given points that lie on a horizontal or vertical line.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	2. state the formula for finding the distance between any two points (x_1, y_1) and (x_2, y_2), using either the differences $x_2 - x_1$ and $y_2 - y_1$ or Δ notation.	100
	3. compute the distance between two points, given their coordinates.	101
	4. use the distance and midpoint formulas in a variety of problems to show that given line segments, triangles and quadrilaterals have (or do not have) specific properties (e.g., show that a quadrilateral is a parallelogram, show that a triangle is a right triangle, show that the length of one line segment is ½ the length of another).	
D. Slope	1. define the slope of a line in terms of the coordinates of two points on the line, using either the differences $x_2 - x_1$ and $y_2 - y_1$ or Δ notation.	110
	2. use the formula $m = \frac{\Delta x}{\Delta y}$ to calculate the slope of a line, given the coordinates of two points on it.	110
	3. show that two parallel lines have equal slopes and that two lines with equal slopes are parallel by counting units of increase or decrease in x and y.	111
	4. determine whether or not two lines are parallel by determining and comparing their slopes.	111
	5. define collinear points.	110

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	6. determine whether or not three given points are collinear by comparing the slopes of the line segments they determine.	111
	7. state that two lines are perpendicular if the product of their slopes is -1, and conversely.	111
	8. determine whether or not two given lines are perpendicular by showing that their slopes do or do not satisfy the above condition	111
	9. use the relationships about the slopes of parallel and perpendicular lines to show that given line segments, triangles, and quadrilaterals have (or do not have) specific properties (e.g., show that a triangle is a right triangle by showing that two of its sides are perpendicular, show that a quadrilateral is a rectangle, square, or rhombus, etc.)	111
E. The Equation of a Line	1. write the equation of any horizontal or vertical line, given the coordinates of one point on the line or a verbal description of its location.	65-66
	2. write the equation of any straight line given its slope and the coordinates of its y-intercept.	65-66
	3. write the equation of any straight line, given the coordinates of any two points on the line.	65-66
	4. determine the slope and y-intercept of any line, given its equation.	
	5. determine, by substitution, whether or not a given point lies on a given line.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XV. AREA	1. state the need for square units to measure the surface of figures.	114
	2. provide examples of the use of areas in real life.	114
	3. state the formulas for finding the areas of rectangles and squares.	117-118
	4. apply the formulas for finding the areas of rectangles and squares in a variety of numerical and algebraic problems, including finding the measures of sides, given the areas.	119
	* 5. prove the thoerem: the area of a parallelogram is the product of its base and altitude.	118
	6. apply the formula for finding the area of a parallelogram to solve a variety of numerical problems, including the use of trigonometry.	119
	* 7. prove the theorem: the area of a triangle is equal to ½ the product of its base and altitude.	118
	8. apply the formula for finding the area of a triangle to solve a variety of numerical problems, including the use of trigonometry.	119
	9. express the area of a right triangle as ½ the product of its legs.	
	10. state the formula for finding the area of an equilateral triangle in terms of the length of a side ($A = \frac{-s^2}{4}\sqrt{3}$).	118

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

11. use the above formulas to solve a variety of numerical problems. 119

* 12. prove the theorem: the area of a trapezoid is equal to ½ the product of its altitude and the sum of its bases (or the product of its median and altitude). 118

13. state the formula for the area of a rhombus in terms of its diagonals ($A = \frac{1}{2}d_1d_2$).

14. apply the above formulas to solve a variety of numerical formulas. 119

15. determine whether or not given triangles are equal in area by applying the following generalizations:

    a. Two triangles have equal areas if they are congruent.

    b. Two triangles have equal areas if their bases and altitudes are congruent.

    c. Two triangles have equal areas if they have the same base and their vertices lie on a line parallel to the base.

16. compare the areas of similar figures, given the measures of a pair of corresponding sides or the ratio of similitude. 121

17. determine the area or a side of a polygon given information about its sides or area and the corresponding sides or area of a similar polygon.

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	18. determine the area of a triangle, given the coordinates of its vertices, by "boxing it in" a rectangle.	120

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
XVI. REGULAR POLYGONS	1. recall the rule for determining the sum of the interior angles of a polygon, given the number of sides.	123
	2. state a rule for determining the sum of the exterior angles of a polygon.	123
	3. define a regular polygon and convex polygon.	124
	4. state a rule for determining the measure of each interior angle of a regular polygon, given the number of sides.	124
	5. state a rule for determining the measure of each exterior angle of a regular polygon, given the number of sides.	124
	6. apply the above rules to a variety of numerical problems about the angles of polygons and regular polygons.	
	7. define circumscribed polygons and inscribed circles.	125
	8. state the theorem: a circle may be inscribed in a regular polygon.	125
	9. state the theorem: a circle may be circumscribed about a regular polygon.	125
	10. define radius, center, apothem, and central angle of a regular polygon.	126
	11. recall the definition of perimeter.	
	12. state the theorem: a radius of a regular polygon bisects its angle.	126

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES

The student will be able to ...

13. state the theorem: if an apothem of a regular polygon is drawn to a side, then it bisects the central angle and it is the perpendicular bisector of the side.

* 14. prove the theorem: the area of a regular polygon is equal to ½ the product of its perimeter and the length of its apothem. 126

15. use the formula $A = \frac{1}{2}ap$ in a variety of numerical problems about regular polygons, including the use of trigonometry. 126

* 16. construct a square inscribed in a given circle. 125

* 17. construct a regular hexagon inscribed in a given circle. 125

* 18. construct an equilateral triangle inscribed in a given circle. 125

* 19. construct a circle inscribed in a given triangle.

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XVII. CIRCUMFERENCE AND AREA OF A CIRCLE	1. define the circumference of a circle.	
	2. describe a circle as the limiting case of a polygon with a great number of sides.	127
	3. calculate better and better approximations of pi by comparing the radii and perimeters of regular polygons of increasing numbers of sides.	127-128
	4. state a formula for the circumference of a circle in terms of pi and a radius or diameter.	128
	5. apply the formulas to find the circumference, radius, or diameter of a circle in a variety of numerical problems.	128
	6. state the formula for the area of a circle.	128
	7. apply the formula for the area of a circle in a variety of numerical problems, including finding the radius, given the area.	128
	8. write a formula for the length of an arc of a circle.	
	9. apply the above formula in solving numerical problems.	
	10. define a sector of a circle.	129
	11. write a formula for finding the area of a sector of a circle.	129
	12. apply the above formula to solve a variety of numerical problems.	129
	13. define a segment of a circle.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	14. draw a diagram relating the areas of a segment, a sector and a triangle.	129
	15. write a formula to describe the relationship between the areas of a segment, a sector and a triangle.	
	16. use the above formula to compute the area of a segment in numerical problems.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XVIII. LOCUS	1. define the locus of points which satisfy a given set of conditions as the figure formed by them.	50
	2. describe a locus as containing <u>every</u> point that satisfies the given set of conditions, and <u>only</u> those points.	50
	3. distinguish between a locus of points and the given figure by drawing them in different colors, as solid and dotted figures, etc.	
	4. describe, verbally, the locus of points a given distance from a given point.	
	5. sketch the locus of points a given distance from a given point.	
	6. describe, verbally, the locus of points a given distance from a given line.	
	7. sketch the locus of points a given distance from a given line.	
	8. describe, verbally, and sketch the locus of points:	50, 51
	a. equidistant from two given points,	70, 130
	b. equidistant from two given intersecting lines (or sides of an angle), or	46, 51, 130
	c. equidistant from two given parallel lines.	130
	9. write the equation of a locus involving horizontal or vertical lines.	65, 131-132

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	10. draw a sketch of the locus of points which satisfy two given sets of conditions by drawing the two separate loci and indicating the point(s) of intersection.	50, 130-133
	11. locate by construction the locus of points equidistant from the vertices of a given triangle (the center of the circumscribed circle) by locating the intersection of the perpendicular bisectors of the sides.	132
	12. locate by construction the locus of points equidistant from the sides of a given triangle (the center of the inscribed circle) by locating the intersection of the bisectors of its angles.	132
	13. write the equation of a line given a verbal description of the relationship between the abscissas and ordinates of its points.	130
	14. express the equation of a line verbally as a locus of points whose abscissas and ordinates satisfy specific conditions.	130

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
XIX. PROVING GEOMETRIC INEQUALITIES	1. write the inverse and contrapositive of a conditional statement.	134-135
	2. state that the inverse of a true statement may be false.	134
	3. state that the contrapositive of a true statement is true (they are logically equivalent).	135
	4. state that, given two quantities, there are only three ways in which they may be related:	137
	a. they are equal,	
	b. the first is greater than the second, or	
	c. the first is less than the second.	
	5. deduce that if a first quantity is greater than a second, and the second is greater than a third, then the first is greater than the third.	135
	6. state the following postulates of inequality:	136-139
	a. Any whole quantity is greater than any of its parts.	
	b. A quantity may be substituted for its equal in any statement of inequality.	
	c. If equal quantities are added to (or subtracted from) unequal quantities, the results are unequal in the same sense.	

CONTENT AREA AND TOPIC	PERFORMANCE OBJECTIVES	REFERENCES
	The student will be able to ...	
	d. If unequal quantities are multiplied by (or divided by) equal positive quantities, the results are unequal in the same sense.	
	e. The sum of the lengths of any two sides of a triangle is greater than the length of the third side.	
	7. prove the theorem: the measure of an exterior angle of a triangle is greater than the measure of either remote interior angle.	137
	8. apply the above theorem to numerical problems.	139
	9. prove the theorem: if two sides of a triangle are unequal, then the angles opposite them are unequal, and the greater angle lies opposite the greater side.	140-142